Electromagnetic Induction
Faraday’s Law

Introduction

Summary of relevant concepts:

- The Faraday’s Law of induction states that an induced electromotive force (emf) is produced by a changing magnetic flux $\Phi$ through a circuit.

- The magnetic flux $\Phi$ is defined as the magnetic induction $B$ through the area of loop $A$:
  \[ \Phi = B \cdot A \cdot \cos \theta \]  
  (1)

  where $\theta$ is the angle between the magnetic induction vector and the normal to the area.

Units for flux are Webers $[Wb = T \cdot m^2 = V \cdot \text{sec}]$

- The magnitude of the induced emf ($\mathcal{E}$) induced in a single loop of wire is equal to the rate of change in the magnetic flux:
  \[ \mathcal{E} = -\frac{\Delta \Phi}{\Delta t} \]  
  (2)

  In this equation the sign minus represents the direction of induced emf according to the Lentz’s law.

- The Lentz’s law states that induced emf produces a current that tends to cancel the changing in magnetic flux. In other words, the induced current in a circuit will produce a magnetic field which opposes the change in the flux that causing the induced current.

- If coil consists of $N$-turns of wire, the emf is given by equation:
  \[ \mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} \]  
  (3)

- A change in flux, and thus the induced emf, can be produced in a coil by a number of methods. For instance, if one moves a bar magnet toward the coil, the number of field lines through the coil will increase increasing the magnetic induction $B$. Thus will cause an increase in flux, and an emf to be induced in the coil. The direction of induced emf depends on the direction of magnetic field lines (the North-end or South-end of magnet moves into the magnet) and on the direction of magnet motion.
• When the bar magnet moves out of the coil, the number of field lines through the coil will decrease causing the decrease in magnetic flux, so the induced current changes the direction on opposite producing its own magnetic field that is supporting the waning magnetic flux through the coil.

• When a magnet is dropped through a coil there is a change in magnetic flux through the coil twice (increasing when the magnet is moving into the coil and decreasing when the magnet is moving out of the coil) thus causing to induce current in the coil with reversing in direction, as shown on a diagram below.

![Diagram](image)

Figure 1. The induced current vs. time diagram when a magnet is dropped through a coil

• The area under the curve \( \mathcal{E} \cdot \Delta t \) represents the change in flux \( \Delta \Phi \) through the coil according to equation (3):

\[
\mathcal{E} \cdot \Delta t = - N \cdot \Delta \Phi
\]  

(4)
The area under the curve could be found by integration. This equation allows defining a magnetic flux and magnetic induction of a magnet.

The goal of the experiment is to measure the electromotive force induced in a coil by a magnet dropping through the coil. You will calculate the area under the curve to find the flux through the coil when the magnet moves in (incoming flux) and the magnet moves out (outgoing flux) to compare the fluxes and calculate the magnetic induction of a magnet.

**Experiment**

**Equipment:**
- Computer Pasco Interface, the Pasco Voltage Sensor, a bar magnets (alnico), a coil, a ruler, connecting wires.

In this experiment, you will plot a graph of the EMF versus time using the Computer Interface and Voltage Sensor. The computer is used as a measurement device which allows making measurements precisely and simultaneously plotting the induced voltage vs. time, as shown on the Figure 1. Induced EMF could be found from the graph by integrating the area under the curve.

![Experimental setup](image)

**Figure 2. The experimental setup**
1. The set up for the experiment is shown on the Figure 2. For this, connect the Voltage Sensor plug into analog interface channel “A”. Attach the Voltage Sensor leads to the coil.

2. Open Pasco Capstone. Click on “Hardware setup”. Look at the hardware; which plug is connected to the sensor? Click on the diagram of the hardware plug and scroll to voltage sensor and select. Click “Hardware setup” again to collapse this window.

3. Choose table and graph for your experiment. Select what data you would like to have displayed.

4. Data recording frequency should be set to 200 Hz. (At bottom of screen)

2. Use the Voltage Sensor to measure the voltage (emf) induced in the coil as a bar magnet moves through the coil. Use Capstone Studio to record, display and analyze the data.

6. Hold the coil so the corner of the coil is beyond the edge of the table and a magnet dropped through the coil can fall free. Make sure that the magnet does not strike the floor, or it may break or be destroyed.

7. Examine and notice which terminal of the coil is connected to the positive (“red”) voltage probe and which one is connected to the negative (“black”) voltage probe.

8. Use a compass or a ‘magnaprobe’ to check which pole of your bar magnet is “North”. Remember that the compass N-end of the arrow shows South-end of the magnet.

Data recording

Step 1. The North-End Magnet Drop

1. Hold the bar magnet with the N pole facing downwards at a distance approximately 2 cm above the induction coil, ready to be dropped through the coil.

2. Start recording data by clicking on the Record icon, and let the magnet drop through the coil freely. Data recording will begin when the magnet falls through the coil and data recording will end automatically after 0.5 seconds after magnet has fallen through the coil and you press the Stop button.
3. Repeat few times to get the best graph for analyzing the data. Save this graph; notice the relative positions of a peak voltages and its height.

4. Set up your graph to show the area under the curve of voltage versus time. For this, click the Sigma (Σ) button and select “area.”

5. In the Graph display, use the cursor to select a rectangle around the first peak of the voltage plot. In DataStudio, the value for ‘Area’ appears in the legend in the Graph. This is the flux through the coil.

6. Record the value of a peak emf as $V_{max}$ and the value of area for the first peak in the Table 1 below.

7. Repeat the process to find the area under the second peak and record the value of integration for the second peak.

8. Compare the results of integration for both peaks by calculating the percent difference between them respect to the average.

$$\% \text{ difference} = \frac{First \ peak \ area - Second \ peak \ area}{Average} \times 100\%$$

Table 1 (North-end drop)

<table>
<thead>
<tr>
<th></th>
<th>$V_{max}$</th>
<th>Area</th>
<th>$%$ difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$V_{sec}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First peak</td>
<td>$\ $</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second peak</td>
<td>$\ $</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aver.</td>
<td>$\ $</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Address these questions in your analysis:**

- Do both peak areas have to be the same or different? Explain your answer.
- What does the peak area represent? Are units for a peak area the same as for flux? Prove it.
- What could be a reason for difference in peak amplitudes $V_{max}$? Why is the first peak smaller and the second peak is higher?
Step 2. The South-End Magnet Drop

1. Now hold the bar magnet with the South end facing downwards at a distance approximately 2 cm above the induction coil, ready to be dropped through the coil.

2. Start recording data, and then drop the magnet through the coil.

3. Observe the graph Voltage versus Time and save a graph noticing the relative positions of a peak voltages and its height.

4. Compare the graph with your experiment in Step 1 when the N-end dropped. Comment your observations. Notice differences and similarities.

5. Find the area under the first peak and record the value of integration for the first peak in a Table 2.

6. Repeat the process to find the area under the second pear and record the value of integration for the second peak in the Table 2.

7. Compare the results of integration for both peaks and calculate the percent difference.

Table 2 (South-end drop)

<table>
<thead>
<tr>
<th>$V_{max}$</th>
<th>Area</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$V$ sec</td>
<td></td>
</tr>
<tr>
<td>First peak</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second peak</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Address these questions in your analysis:
- Is the maximum voltage different for both peaks now? Which is higher? Why?
- Are the incoming and outgoing fluxes different for both peaks now?
- Compare the results of integration with Step 1: if there is any significant difference in peak areas when S-end or N-end facing down when magnet is moving through the coil?
Step 3. Flipping the Coil

Flip the coil and drop the magnet with the South–end go first from the same distance which is about 2 cm above the coil.
Go over the data recording and data analyzing procedure as in the Step 2.

Save a graph, showing the relative positions of a peak voltages and its height and compare it with the previous graph in Step 2.

- Does the graph have the similar appearance as in the Step 2 experiment with the same South-end drop? What is the difference? How does flipping the coil affect the results?

- Record the results of integration for both peaks in the Table 3 and calculate the percent difference between the peaks.

Table 3. Flipping the coil

<table>
<thead>
<tr>
<th></th>
<th>$V_{max}$</th>
<th>Area</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V$</td>
<td>$V$ sec</td>
<td></td>
</tr>
<tr>
<td>First peak</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second peak</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Compare the results of your experiment in Step 3 with two previous experiments in Step 1 and Step 2.
- Is the maximum voltage different for both peaks now?
- Are incoming and outgoing fluxes affected by flipping the coil?
- Why the peaks are in opposite direction for each of the experiment?
Step 4. Changing the Height of Drop

With the same magnet and coil orientation as in the Step 3, drop the magnet through the coil freely from the higher distance about 5 cm above the coil. Go over data recording and data analyzing procedure as in previous experiments. Save a graph emf vs. time and compare it with the previous graph in the Step 3.

Record the results of measurement and calculation in the Table 4 and compare the results with previous experiment.

Table 4 (A higher height drop)

<table>
<thead>
<tr>
<th></th>
<th>V&lt;sub&gt;max&lt;/sub&gt;</th>
<th>Area</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V</td>
<td>V sec</td>
<td></td>
</tr>
<tr>
<td>First peak</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Second peak</td>
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<tr>
<td>Av. =</td>
<td></td>
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</tbody>
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- How does the peak voltage change comparing with the previous experiment? Why does a drop height affect a peak voltage?
- Does the flux change comparing with the previous experiment?

Analysis:

- As the same bar magnet was used in each of the experiment in the Steps 1 through the Step 4, you can evaluate the magnetic induction B of the bar magnet. Combining the equation 4 and equation 1 you have for B:

\[
B = \frac{(\varepsilon \Delta t)}{NA}
\]  

(5)
In this equation, \((\varepsilon \Delta t)\) is “the area under the curve” which is equivalent to the flux through the coil, \(N\) is the number of turns in the coil, and \(A\) is the cross section area of the coil.

When calculating magnetic induction \(B\) of the bar magnet, use the average values of the “areas under the curve” \((\varepsilon \Delta t)\) from your experiments.

Examine the coil to find out the number of turns \(N\):

\[
N = \rule{5cm}{0.1pt}
\]

Measure the diameter \(d\) of the coil to calculate its cross section area \(A\).

\[
d = \rule{5cm}{0.1pt}
\]

\[
A = \frac{\pi d^2}{4} = \rule{5cm}{0.1pt}, \text{ m}^2
\]

Represent the results of calculation in a Table 5.

<table>
<thead>
<tr>
<th>Table 5</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Av. Area</td>
<td>B</td>
<td>ΔB</td>
</tr>
<tr>
<td>V sec</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>Step 1</td>
<td></td>
<td></td>
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<tr>
<td>Step 2</td>
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<tr>
<td>Step 3</td>
<td></td>
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<tr>
<td>Step 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B_{av})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the table above, \(\Delta B = B_i - B_{av}\) is a deviation of a measured value \(B_i\) from the average, where \(i\) is a number of step which varies from 1 to 4 in your experiment;

\[
\sigma = \frac{\sqrt{2\sum(\Delta B_i)^2}}{n(n-1)} \text{ where } n = 4.
\]

Represent the result of your experiment as: \(B = B_{av} \pm \sigma_m\)