

Bring out your dead!

How quickly does disease spread through a population? We will create a mathematical model to investigate the spread of disease using the Kermack-McKendrick model, also known as “SIR” and found in section 2.1 of our text. This model assumes that the entire population can be separated into three distinct groups

- **Susceptible.** Individuals who haven't caught the disease yet.
- **Infected.** Individuals who have the disease
- **Recovered** The survivors!

Each time period, some of the **Susceptible** become **Infected** (they catch the disease), and some **Infected** join the **Recovered** group (they recover). The rate at which the susceptible population becomes infected depends on **Contact Probability**, which is the chance a particular susceptible is exposed to a particular **Infected**. The number of new infected each period is given by:

$$S \rightarrow I = S * (1 - (1 - \text{Contact Probability})^I)$$

The rate at which the infected population recovers depends on the **Healing rate**. This is the fraction of the infected population which recovers each time period.

$$I \rightarrow R = I * \text{Healing Rate}$$

In your initial model, start with the following values:

- Initial population = 1000
- Contact Probability = 0.002 (0.2%)
- Healing Rate = 1
- Initial Infectives = 3 (implies initial Susceptibles = ?)

1. Construct a graph that simultaneously shows the population in each group (Susceptible, Infected and Recovered). Make sure the graph covers enough time for the epidemic to settle down.

Additional Questions

2. Introduce scroll bars for Contact Rate (0 to 0.5, steps of 0.001) and Healing Rate (0 to 1, steps of 0.01). [See how changing these affect the population.]
3. Change the model so that the population increases over time (there are births!). A number of new citizens, proportional to total population, are added to the susceptible population each period. Start with a population growth rate of 0.008 per period (so $0.008 * (\text{Total Pop})$ are added to **S** in the next step). [This growing population can produce waves of epidemic relapses. You may have to greatly increase the time period covered in the graph to see the waves.]
4. Along with the population growth, suppose that the disease is sometimes fatal. Now each period, some infected recover and some die ☹️ (don't forget, some are still sick); Create a model that will allow you to observe fatality rates that result in a significant drop (try for 80%) in population before the population begins to recover. [Make sure your $(\text{Healing Rate} + \text{Death Rate}) \leq 1$, otherwise we would have some people both recovering and dying.]
5. Imagine there are two island nations. At first, everyone lives on the first island, Ni. However, once an individual comes down with the disease, they are forced to move to Shrubbery Island (i.e. they have one period to spread the disease and are forced to move) Assume the populations of both islands have population growth (as before) at a rate of 0.008. [Vary Contact, Heal and Death rates to see the long term effects on the island populations.]