COMAP's Mathematical Contest in Modeling – past problems

The Stunt Person (2003)

An exciting action scene in a movie is going to be filmed, and you are the stunt coordinator! A stunt person on a motorcycle will jump over an elephant and land in a pile of cardboard boxes to cushion their fall. You need to protect the stunt person, and also use relatively few cardboard boxes (lower cost, not seen by camera, etc.).

Your job is to:

- determine what size boxes to use
- determine how many boxes to use
- determine how the boxes will be stacked
- determine if any modifications to the boxes would help
- generalize to different combined weights (stunt person & motorcycle) and different jump heights

An example: watch Robbie (son of Evel) Knievel jump a bunch of Coca – Cola trucks.

Take a Bath (2008)

Consider the effects on land from the melting of the north polar ice cap due to the predicted increase in global temperatures. Specifically, model the effects on the coast of Florida every ten years for the next 50 years due to the melting, with particular attention given to large metropolitan areas. Propose appropriate responses to deal with this. A careful discussion of the data used is an important part of the answer.

Parking Lot Design (1987)

The owner of a paved, 100' by 200' , corner parking lot in a New England town hires you to design the layout, that is, to design how the ``lines are to be painted.'' You realize that squeezing as many cars into the lot as possible leads to right-angle parking with the cars aligned side by side. However, inexperienced drivers have difficulty parking their cars this way, which can give rise to expensive insurance claims. To reduce the likelihood of damage to parked vehicles, the owner might then have to hire expert drivers for ``valet parking.'' On the other hand, most drivers seem to have little difficulty in parking in one attempt if there is a large enough ``turning radius'' from the access lane. Of course, the wider the access lane, the fewer cars can be accommodated in the lot, leading to less revenue for the parking lot owner.

A Faster QuickPass System (2004)

"QuickPass" systems are increasingly appearing to reduce people's time waiting in line, whether it is at tollbooths, amusement parks, or elsewhere. Consider the design of a QuickPass system for an amusement park. The amusement park has experimented by offering QuickPasses for several popular rides as a test. The idea is that for certain popular rides you can go to a kiosk near that ride and insert your daily park entrance ticket, and out will come a slip that states that you can return to that ride at a specific time later. For example, you insert your daily park entrance ticket at 1:15 pm, and the QuickPass states that you can come back between 3:30 and 4:30 pm when you can use your slip to enter a second, and presumably much shorter, line that will get you to the ride faster. To prevent people from obtaining QuickPasses for several rides at once, the QuickPass machines allow you to have only one active QuickPass at a time.

You have been hired as one of several competing consultants to improve the operation of QuickPass. Customers have been complaining about some anomalies in the test system. For example, customers observed that in one instance QuickPasses were being offered for a return time as long as 4 hours later. A short time later on the same ride, the QuickPasses were given for times only an hour or so later. In some instances, the lines for people with Quickpasses are nearly as long and slow as the regular lines.

The problem then is to propose and test schemes for issuing QuickPasses in order to increase people's enjoyment of the amusement park. Part of the problem is to determine what criteria to use in evaluating alternative schemes. Include in your report a non-technical summary for amusement park executives who must choose between alternatives from competing consultants.

Gerrymandering (2007)

The United States Constitution provides that the House of Representatives shall be composed of some number (currently 435) of individuals who are elected from each state in proportion to the state's population relative to that of the country as a whole. While this provides a way of determining how many representatives each state will have, it says nothing about how the district represented by a particular representative shall be determined geographically. This oversight has led to egregious (at least some people think so, usually not the incumbent) district shapes that look "unnatural" by some standards.

Hence the following question: Suppose you were given the opportunity to draw congressional districts for a state. How would you do so as a purely "baseline" exercise to create the "simplest" shapes for all the districts in a state? The rules include only that each district in the state must contain the same population. The definition of "simple" is up to you; but you need to make a convincing argument to voters in the state that your solution is fair. As an application of your method, draw geographically simple congressional districts for the state of New York.

Snowboard Course (2011)

Determine the shape of a snowboard course (currently known as a “halfpipe”) to maximize the production of “vertical air” by a skilled snowboarder.

"Vertical air" is the maximum vertical distance above the edge of the halfpipe.

Tailor the shape to optimize other possible requirements, such as maximum twist in the air.

What tradeoffs may be required to develop a “practical” course?

Grade Inflation (1998)

Some college administrators are concerned about the grading at A Better Class (ABC) college. On average, the faculty at ABC have been giving out high grades (the average grade now given out is an A-), and it is impossible to distinguish between the good and mediocre students. The terms of a very generous scholarship only allow the top 10% of the students to be funded, so a class ranking is required.

The dean had the thought of comparing each student to the other students in each class, and using this information to build up a ranking. For example, if a student obtains an A in a class in which all students obtain an A, then this student is only ``average'' in this class. On the other hand, if a student obtains the only A is a class, then that student is clearly ``above average.'' Combining information from several classes might allow students to be placed in deciles (top 10%, next 10%, etc.) across the college.

Assuming that the grades given out are (A+, A, A-, B+,...), can the dean's idea be made to work? Assuming that the grades given out are only (A,B,C,...), can the dean's idea be made to work? Can any other schemes produce a desired ranking? A concern is that the grade in a single class could change many student's deciles. Is this possible?

Teams should design data sets to test and demonstrate their algorithms. Teams should characterize data sets that limit the effectiveness of their algorithms.

The Sweet Spot (2010)

Explain the “sweet spot” on a baseball bat.

Every hitter knows that there is a spot on the fat part of a baseball bat where maximum power is transferred to the ball when hit. Why isn’t this spot at the end of the bat? A simple explanation based on torque might seem to identify the end of the bat as the sweet spot, but this is known to be empirically incorrect. Develop a model that helps explain this empirical finding.

Some players believe that “corking” a bat (hollowing out a cylinder in the head of the bat and filling it with cork or rubber, then replacing a wood cap) enhances the “sweet spot” effect. Augment your model to confirm or deny this effect. Does this explain why Major League Baseball prohibits “corking”?

Does the material out of which the bat is constructed matter? That is, does this model predict different behavior for wood (usually ash) or metal (usually aluminum) bats? Is this why Major League Baseball prohibits metal bats?

Unlawful Assembly (1999)

Many public facilities have signs in rooms used for public gatherings which state that it is "unlawful" for the rooms to be occupied by more than a specified number of people. Presumably, this number is based on the speed with which people in the room could be evacuated from the room's exits in case of an emergency. Similarly, elevators and other facilities often have "maximum capacities" posted.

Develop a mathematical model for deciding what number to post on such a sign as being the "lawful capacity". As part of your solution discuss criteria, other than public safety in the case of a fire or other emergency, that might govern the number of people considered "unlawful" to occupy the room (or space). Also, for the model that you construct, consider the differences between a room with movable furniture such as a cafeteria (with tables and chairs), a gymnasium, a public swimming pool, and a lecture hall with a pattern of rows and aisles. You may wish to compare and contrast what might be done for a variety of different environments: elevator, lecture hall, swimming pool, cafeteria, or gymnasium. Gatherings such as rock concerts and soccer tournaments may present special conditions.

Apply your model to one or more public facilities at your institution (or neighboring town). Compare your results with the stated capacity, if one is posted. If used, your model is likely to be challenged by parties with interests in increasing the capacity. Write an article for the local newspaper defending your analysis.

Designing a Traffic Circle (2009)

Many cities and communities have traffic circles—from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road that gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a Traffic Engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

Airline Overbooking (2002)

You're all packed and ready to go on a trip to visit your best friend in New York City. After you check in at the ticket counter, the airline clerk announces that your flight has been overbooked. Passengers need to check in immediately to determine if they still have a seat.

Historically, airlines know that only a certain percentage of passengers who have made reservations on a particular flight will actually take that flight. Consequently, most airlines overbook-that is, they take more reservations than the capacity of the aircraft. Occasionally, more passengers will want to take a flight than the capacity of the plane leading to one or more passengers being bumped and thus unable to take the flight for which they had reservations. Airlines deal with bumped passengers in various ways. Some are given nothing, some are booked on later flights on other airlines, and some are given some kind of cash or airline ticket incentive.

Consider the overbooking issue in light of the current situation:

- Less flights by airlines from point A to point B
- Heightened security at and around airports
- Passengers' fear
- Loss of billions of dollars in revenue by airlines to date

Build a mathematical model that examines the effects that different overbooking schemes have on the revenue received by an airline company in order to find an optimal overbooking strategy, i.e., the number of people by which an airline should overbook a particular flight so that the company's revenue is maximized. Insure that your model reflects the issues above, and consider alternatives for handling "bumped" passengers. Additionally, write a short memorandum to the airline's CEO summarizing your findings and analysis.

The Airplane Seating Problem (2007)

Airlines are free to seat passengers waiting to board an aircraft in any order whatsoever. It has become customary to seat passengers with special needs first, followed by first-class passengers (who sit at the front of the plane). Then coach and business-class passengers are seated by groups of rows, beginning with the row at the back of the plane and proceeding forward. Apart from consideration of the passengers' wait time, from the airline's point of view, time is money, and boarding time is best minimized. The plane makes money for the airline only when it is in motion, and long boarding times limit the number of trips that a plane can make in a day. The development of larger planes, such as the Airbus A380 (800 passengers), accentuate the problem of minimizing boarding (and deboarding) time.

Devise and compare procedures for boarding and deboarding planes with varying numbers of passengers: small (85–210), midsize (210–330), and large (450–800). Prepare an executive summary, not to exceed two single-spaced pages, in which you set out your conclusions to an audience of airline executives, gate agents, and flight crews.

An article appeared in the NY Times Nov 14, 2006 addressing procedures currently being followed and the importance to the airline of finding better solutions. The article can be seen at:
http://travel2.nytimes.com/2006/11/14/business/14boarding.html

Tollbooths (2005)

Heavily-traveled toll roads such as the Garden State Parkway , Interstate 95, and so forth, are multi-lane divided highways that are interrupted at intervals by toll plazas. Because collecting tolls is usually unpopular, it is desirable to minimize motorist annoyance by limiting the amount of traffic disruption caused by the toll plazas. Commonly, a much larger number of tollbooths is provided than the number of travel lanes entering the toll plaza. Upon entering the toll plaza, the flow of vehicles fans out to the larger number of tollbooths, and when leaving the toll plaza, the flow of vehicles is required to squeeze back down to a number of travel lanes equal to the number of travel lanes before the toll plaza. Consequently, when traffic is heavy, congestion increases upon departure from the toll plaza. When traffic is very heavy, congestion also builds at the entry to the toll plaza because of the time required for each vehicle to pay the toll.

Make a model to help you determine the optimal number of tollbooths to deploy in a barrier-toll plaza. Explicitly consider the scenario where there is exactly one tollbooth per incoming travel lane. Under what conditions is this more or less effective than the current practice? Note that the definition of "optimal" is up to you to determine.