Math 105

Introduction to Graphs

Section 9.1

What is a graph? A graph is a set of points called vertices and line segments or arcs called edges that connect vertices.

The number of edges that meet at a vertex is called the degree of a vertex.
A graph is connected if for every pair of vertices there is a path connecting the two.
A graph is complete if every possible edge is drawn between vertices.

Examples of graphs:

Equivalent Graphs - When are two graphs equivalent (or isomorphic)?

Example 1: Are the 2 graphs below equivalent (isomorphic)?

Example 2: Are the 2 graphs below equivalent (isomorphic)?

For additional practice of determining if two graphs are equivalent go to:
**Euler Circuits**

**Example 3:** In the eighteenth century, the Pregel River in a city called Konigsberg (located in modern-day Russia, now called Kaliningrad) surrounded an island before splitting into two. Seven bridges crossed the river and connected four different land areas, similar to the map drawn below.

![Map of Konigsberg](image1)

Many citizens of the time attempted to take a stroll that would lead them across each bridge and return them to the starting point without traversing the same bridge twice. Can you do it? So is it impossible to take such a stroll? Or is it just a difficult task?

Convert the geographical situation above to a graph with edges and vertices where each land area is represented by a vertex and connect two vertices if there is a bridge between the land areas.

![Graph of Konigsberg](image2)

Question: Can we start at any vertex, move through each edge once (but not more than once), and return to the starting vertex? If so, then we call this an **Eulerian circuit**.

**Example 4:** If we can find an Eulerian circuit in a graph, what conditions must be true in order to find such a circuit?

![Graphs](image3)

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**Example 5:** The map below gives the territory of a parking control officer. The dots represent meters that must be checked. Draw the graph that would be useful for finding an efficient route.

![Map of parking control officer's territory]

**Example 6:** Give three real world applications in which a worker would want to find an Euler circuit on a street network.

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**Euler Walks**

An Euler walk does not require that we start and stop at the same vertex, whereas an Euler circuit does.

**Example 7:** Does the Konigsberg bridge problem have a solution if we did not need to return to the starting point? Try it!

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**General results for when graphs have an Euler circuit and an Euler walk:**

1. 

2. 

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For additional practice with Eulerian Graphs go to: 
Hamiltonian Circuits
Suppose we want to find a route that visits each vertex once and only once. This time we don’t care if we use all the edges or not.

Example 8: Does the graph below contain a Hamiltonian circuit?

Example 9: The floor plan of an art gallery is below.

Example 10:

a) Construct an example of a graph with no Hamiltonian circuit.
b) Construct an example of a connected graph that has an Euler circuit, but no Hamiltonian circuit.
c) Construct an example of a connected graph that has a Hamiltonian circuit but does not have an Euler circuit.
d) The route of a neighborhood garbage truck generally follows an Euler circuit. Under what circumstances should it instead follow a Hamiltonian circuit?

For additional practice with Hamiltonian Graphs go to: