

Example 1. S.W. Laagakos and F. Mosteller of Harvard University fed mice different doses of red dye number 40 and recorded the time of death in weeks. Results for female mice, dosage and time of death are shown in the data.

- X1 = time of death for mice in control group
- X2 = time of death for mice in group with low dosage
- X3 = time of death for mice in group with medium dosage
- X4 = time of death for mice in group with high dosage

| X1 | X2 | X3 | X4 |
|-----|----|----|-----|
| 70 | 49 | 30 | 34 |
| 77 | 60 | 37 | 36 |
| 83 | 63 | 56 | 48 |
| 87 | 67 | 65 | 48 |
| 92 | 70 | 76 | 65 |
| 93 | 74 | 83 | 91 |
| 100 | 77 | 87 | 98 |
| 102 | 80 | 90 | 102 |
| 102 | 89 | 94 | |
| 103 | 97 | | |
| 96 | | | |

So the parameters to be compared are as follows:

- μ_1 = average time of death for control group
- μ_2 = average time of death for group with low dosage
- μ_3 = average time of death for group with medium dosage
- μ_4 = average time of death for group with high dosage

Null Hypothesis H_0 : _____

Alternative Hypothesis H_a : _____

F test statistic: _____ P-value: _____

Since _____, we _____ the null hypothesis.

Conclusion (in plain English):

Example 2. The following data was obtained from the National Transportation Safety Administration. New cars were purchased and crashed into a fixed barrier at 35 mph, and the listed measurements were recorded for the dummy in the driver's seat. The chest deceleration data are given below, broken into groups by size of car. *Does the data indicate that the different weight categories have the same mean crash deceleration?*

The parameters to be compared are as follows:

- μ_1 = average _____ among _____
- μ_2 = average _____ among _____
- μ_3 = average _____ among _____
- μ_4 = average _____ among _____

| | | | | | |
|------------|----|----|----|----|----|
| Subcompact | 55 | 47 | 59 | 49 | 42 |
| Compact | 57 | 57 | 46 | 54 | 51 |
| Midsize | 45 | 53 | 49 | 51 | 46 |
| Full-size | 44 | 45 | 39 | 58 | 44 |

Null Hypothesis H_0 : _____

Alternative Hypothesis H_a : _____

F test statistic: _____ P-value: _____

Since _____, we _____ the null hypothesis.

Conclusion (in plain English):

Example 3. An experiment was conducted to study the reaction effects of four drugs on a nervous disorder. Subjects with the nervous disorder were independently and randomly assigned to the four drug groups. The reaction times for an experimental task were recorded for the 25 subjects after they were administered the drug.

| Drug 1 | Drug 2 | Drug 3 | Drug 4 |
|--------|--------|--------|--------|
| 3 | 5 | 6 | 2 |
| 5 | 7 | 5 | 4 |
| 4 | 3 | 7 | 3 |
| 6 | 4 | 9 | 4 |
| 4 | 5 | 6 | 2 |
| | 3 | 7 | 5 |
| | 6 | 8 | |

Test at the 5% level the hypothesis of equality of mean reaction times for the four drugs.

The parameters to be compared are as follows:

- μ_1 = average _____ among _____
- μ_2 = average _____ among _____
- μ_3 = average _____ among _____
- μ_4 = average _____ among _____

Null Hypothesis H_0 : _____

Alternative Hypothesis H_a : _____

F test statistic: _____ P-value: _____

Since _____, we _____ the null hypothesis.

Conclusion (in plain English):

Example 4. Suppose the USGA wants to compare the mean distances associated with four different brands of golf balls when struck with a driver. A completely randomized design is used, with Iron Byron, the USGA's robotic golfer. Iron Byron uses a driver to hit a random sample of 10 balls of each brand in a random sequence. The distance is recorded for each hit, and the results are shown by brand in the following table. Test at the 5% significance level if there is a difference in the brands.

| Brand A | Brand B | Brand C | Brand D |
|---------|---------|---------|---------|
| 251.2 | 263.2 | 269.7 | 251.6 |
| 245.1 | 262.9 | 263.2 | 248.6 |
| 248.0 | 265.0 | 277.5 | 249.4 |
| 251.1 | 254.5 | 267.4 | 242.0 |
| 260.5 | 264.3 | 270.5 | 246.5 |
| 250.0 | 257.0 | 265.5 | 251.3 |
| 253.9 | 262.8 | 270.7 | 261.8 |
| 244.6 | 264.4 | 272.9 | 249.0 |
| 254.6 | 260.6 | 275.6 | 247.1 |
| 248.8 | 255.9 | 266.5 | 245.9 |

The parameters to be compared are as follows:

- μ_1 = average _____ among _____
- μ_2 = average _____ among _____
- μ_3 = average _____ among _____
- μ_4 = average _____ among _____

Null Hypothesis H_0 : _____

Alternative Hypothesis H_a : _____

F test statistic: _____ P-value: _____

Since _____, we _____ the null hypothesis.

Conclusion (in plain English):

Example 1. The parameters being compared are...

- μ_1 = average time of death for control group
- μ_2 = average time of death for group with low dosage
- μ_3 = average time of death for group with medium dosage
- μ_4 = average time of death for group with high dosage

Null Hypothesis H_0 : $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (That is, there is no association between red dye consumption and life span in mice.)
Alternative Hypothesis H_a : Not all the means are equal. (That is, there is an association between red dye consumption and life span in mice.)

Results: F test statistic = 3.62 P-value = 0.0227

Since the P-value is small, we reject the null hypothesis. That is, we have statistically significant evidence that there is an association between red dye consumption and life span in mice.

Example 2. The parameters being compared are...

- μ_1 = average chest deceleration for all subcompact cars
- μ_2 = average chest deceleration for all compact cars
- μ_3 = average chest deceleration for all midsized cars
- μ_4 = average chest deceleration for all full-sized cars

Null Hypothesis H_0 : $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (That is, there is no association between car size and chest deceleration.)

Alternative Hypothesis H_a : Not all the means are equal. (That is, there is an association between car size and chest deceleration.)

Results: F test statistic = 1.34 P-value = 0.296

Since the P-value is **not** small, we **do not** reject the null hypothesis. That is, we **do not** have statistically significant evidence that there is an association between car size and chest deceleration.

Note. *The calculator does not give you enough information to determine if the data meets the assumptions of the ANOVA test. We will not dwell on this in MAT117 but it is an important consideration of performing this kind of test in a real setting. In Example 1, for example, lifespan measurements are very likely skewed left and we are not making up for it with a large sample size; moreover, the Group 4 sample standard deviation is well over twice the Group 1 sample standard deviation. Example 2 has similar challenges to the assumptions of ANOVA. Examples 3 and 4 meet the assumptions much better, which can be seen when doing the ANOVA test in a statistical package such as StatCrunch.*

Example 3. The parameters being compared are...

- μ_1 = average reaction time among all patients on Drug 1
- μ_2 = average reaction time among all patients on Drug 2
- μ_3 = average reaction time among all patients on Drug 3
- μ_4 = average reaction time among all patients on Drug 4

Null Hypothesis H_0 : $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (That is, there is no association between which drug one takes and ones reaction time.)

Alternative Hypothesis H_a : Not all the means are equal. (That is, there is an association between which drug one takes and ones reaction time.)

Results: F test statistic = 8.18 P-value = 0.0009

Since the P-value is small, we reject the null hypothesis. That is, we have statistically significant evidence that there is an association between which drug one takes and ones reaction time.

Example 4. The parameters being compared are...

- μ_1 = average distance a golf ball is hit among all Brand A clubs
- μ_2 = average distance a golf ball is hit among all Brand B clubs
- μ_3 = average distance a golf ball is hit among all Brand C clubs
- μ_4 = average distance a golf ball is hit among all Brand D clubs

Null Hypothesis H_0 : $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (That is, there is no association between golf club brand and distance it can hit a ball.)

Alternative Hypothesis H_a : Not all the means are equal. (That is, there is an association between golf club brand and the distance it can hit a ball.)

Results: F test statistic = 43.99 P-value less than 0.0001

Since the P-value is small, we reject the null hypothesis. That is, we have statistically significant evidence that there is an association between golf club brand and the distance it can hit a ball.