Part I. For each $\mathbf{b}$ and S, determine if $\mathbf{b}$ can be written as a linear combination of the vectors in the set S. If yes, explicitly show the linear combination that is equal to $\mathbf{b}$.

1. $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $S = \left\{ \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right\}$

2. $\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$, $S = \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} \right\}$

3. $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$, $S = \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \right\}$

4. $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$, $S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} \right\}$

Part II. For each matrix A below, determine if the columns of A span $\mathbb{R}^4$. If they do not, find an example of a vector $\mathbf{b}$ for which the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent:

1. $A = \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$

2. $A = \begin{bmatrix} 5 & -7 & -4 & 9 \\ 6 & -8 & -7 & 5 \\ 4 & -4 & -9 & -9 \\ -9 & 11 & 16 & 7 \end{bmatrix}$

3. $A = \begin{bmatrix} 12 & -7 & 11 & -9 & 5 \\ -9 & 4 & -8 & 7 & -3 \\ -6 & 11 & -7 & 3 & -9 \\ 4 & -6 & 10 & -5 & 12 \end{bmatrix}$

4. $A = \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$

Matrices for problems 17-20 in Section 1.4

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$