

1. Determine in each case if the given vectors are linearly independent. In each case there is a simple explanation “why.”

$$(a) \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

$$(b) \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$$

2. Determine in each case if the columns of the matrix form a linearly independent set. If they do not, give a non-trivial linear combination of the column vectors that is equal to $\mathbf{0}$.

$$(a) A = \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{bmatrix}$$

3. Mark each of the following statements as True or False. A False answer should be accompanied by a counterexample.

- (a) If $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent, and $\mathbf{w} \in \text{Span}\{\mathbf{u}, \mathbf{v}\}$, the $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent.
- (b) The columns of an 4×5 matrix are linearly dependent.
- (c) If a set of vectors in \mathbb{R}^n is linearly dependent, then the set contains more than n vectors.

4. Find all values of h for which the set $\left\{ \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix} \right\}$ is linearly dependent.