

Give a proof or counterexample to each statement. Use the *definition* of linear independence/dependence.

1. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are vectors in \mathbb{R}^4 and $\mathbf{v}_3 = \mathbf{v}_1 - \mathbf{v}_4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.
2. If \mathbf{v}_1 and \mathbf{v}_2 are vectors in \mathbb{R}^4 and \mathbf{v}_2 is *not* a scalar multiple of \mathbf{v}_1 , then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly independent.
3. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are vectors in \mathbb{R}^4 and \mathbf{v}_3 is *not* a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.
4. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are vectors in \mathbb{R}^4 and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.
5. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are vectors in \mathbb{R}^4 and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.