1. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $T(\mathbf{x}) = A\mathbf{x}$, is supposed to reflect an image about the line $y = x$.

   (a) Draw a picture to explain why $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}$, and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$.

   (b) Find the matrix A for this linear transformation. Use the Renault applet to verify that it really does to the picture what it is supposed to do.

2. For each of the linear transformations described below, determine if its (a) one-to-one and/or (b) onto. A "no" answer should be accompanied by an example to support the answer.

   (a) The "reflection" transformation T described above.

   (b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$

   (c) $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} 7 & 5 & 4 & -9 \\ 10 & 6 & 16 & -4 \\ 12 & 8 & 12 & 7 \\ -8 & -6 & -2 & 5 \end{bmatrix}$.