

1. Determine all values of h for which the matrix below is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \xrightarrow{-5R_1+R_2} \begin{bmatrix} 1 & -3 & -2 \\ 0 & 15+h & 3 \end{bmatrix}$$

This will be inconsistent if bottom row is $[0 \ 0 \ 3]$ & consistent otherwise. Therefore, inconsistent if and only if $h = -15$, so the system is consistent if and only if $h \neq -15$.

2. Mark each of the following statements as True or False:

- F (a) The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.
- T (b) Whenever a system has free variables, the solution set contains many solutions.
- T (c) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.
- F (d) If one row in an echelon form of an augmented matrix is $[0 \ 0 \ 0 \ 5 \ 0]$, then the associated linear system is inconsistent.

3. Write a system of equations that is equivalent to the following vector equation. (Read Example 5 of Section 1.3)

$$x_1 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 8x_2 + x_3 = 0$$

$$3x_1 + 5x_2 - 6x_3 = 0$$

4. Determine if the vector $\begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$.

(Read Example 5 of Section 1.3. Use technology to do the necessary row reductions!)

Vector Equation: $x_1 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$, which is same as system of equations

$$\begin{cases} x_1 + 2x_3 = -5 \\ -2x_1 + 5x_2 = 11 \\ 2x_1 + 5x_2 + 8x_3 = -7 \end{cases} \leftarrow \text{Consistent?}$$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 4/5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is inconsistent, so **NO**