1. Prove that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in the Span $\left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ for all values of h and k.

$$\begin{bmatrix} 2 & 2 & h \\ -1 & 1 & k \end{bmatrix} \to \begin{bmatrix} -1 & 1 & k \\ 2 & 2 & h \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -k \\ 2 & 2 & h \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -k \\ 0 & 4 & h-2k \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -1 & -k \\ 0 & 1 & \frac{h-2k}{4} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -k+\frac{h-2k}{4} \\ 0 & 1 & \frac{h-2k}{4} \end{bmatrix}$$, which is consistent regardless of the value of $h + k$.

2. Mark each of the following statements as True or False:

T (a) An example of a linear combination of vectors $\mathbf{u}$ and $\mathbf{v}$ is $\frac{1}{2}\mathbf{u}$.

T (b) The solution set of the linear system whose augmented matrix is $[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{b}]$ is the same as the solution set to the vector equation $x_1\mathbf{a}_1 + x_1\mathbf{a}_2 + x_1\mathbf{a}_3 = \mathbf{b}$.

F (c) The weights $c_1, c_2, \ldots, c_p$ in a linear combination $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p$ cannot all be zero.

T (d) If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^m$.

3. Write the system of equations below first as a vector equation and then as a matrix equation.

$$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 - 3x_2 &= 2 \end{aligned}$$

Vector equation

$$x_1 \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Matrix equation

$$\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

4. Determine if every vector in $\mathbb{R}^4$ can be written as a linear combination of the vectors in the matrix M below:

$$M = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

No

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the last row is "all 0s", there are ~~can be~~ vectors b for which Mx = b have no solution.