MAT318
Elementary Linear Algebra

Name: Solution

Quiz 2

1. **(4 points)** Write the system of equations below as an equivalent vector equation:

$$\begin{array}{rcrcrcr} 2x & - & 4y & & & = & 10 \\ & & 2y & + & 2z & = & -2 \\ x & & & + & 2z & = & 3 \end{array}$$

$$x\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + y\begin{bmatrix} -4 \\ 2 \\ 0 \end{bmatrix} + z\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 3 \end{bmatrix}$$

2. **(6 points)** Determine if vector **b** can be written as a linear combination of the vectors formed from the columns of the matrix A. If so, give a specific linear combination of the vectors that is equal to b; if not, explain why not. Give a complete explanation using sentences in either case. The row reduced augmented matrix (A augmented with the vector **b**) is given at the bottom of the page.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -6 \end{bmatrix}$$

The rref matrix below tells us that $x_1 = -\frac{26}{3}x_3 - \frac{19}{3}$, $x_2 = -\frac{5}{3}x_3 - \frac{7}{3}$, with x_3 free. A particular solution is $x_3 = 1$, $x_1 = -15$, $x_2 = -4$, so we conclude that $\vec{b}$ can be written as $-15\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} - 4\begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$, which is easy to check as correct.

$$[A \;\; \mathbf{b}] = \begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -6 \end{bmatrix} \overset{\text{RREF}}{\longrightarrow} \begin{bmatrix} 1 & 0 & 26/3 & -19/3 \\ 0 & 1 & 5/3 & -7/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$