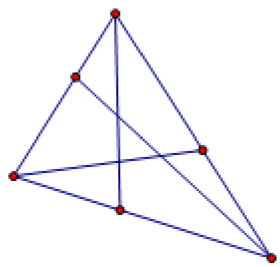
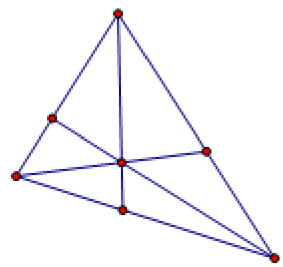
Presentation: Ceva's Theorem, Part 1

A line segment connecting a vertex of a triangle to a point on the opposite side is called a *cevian*. The *medians* of a triangle – the line segments connecting the vertices of the triangle to the midpoints of the opposite side – are important examples of cevians. It is a fact that the three medians of a triangle intersect at a single point, called the *centroid* of the triangle.

In the diagrams below, cevians are shown. In the triangle on the right, the cevians all intersect at a single point; in this case we say that the cevians are **concurrent** at that point. In the diagram on the right, the cevians are not concurrent. Ceva's Theorem (after whom cevians are named) tells us the conditions under which cevians are concurrent.



non-concurrent cevians

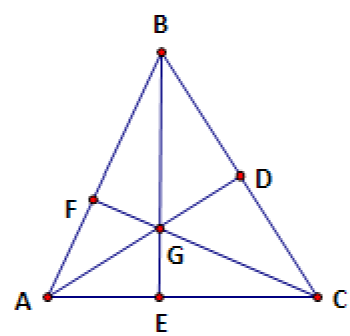


concurrent cevians

Open the "Ceva's Theorem" Sketchpad file located on the course webpage to see the statement of Ceva's Theorem.

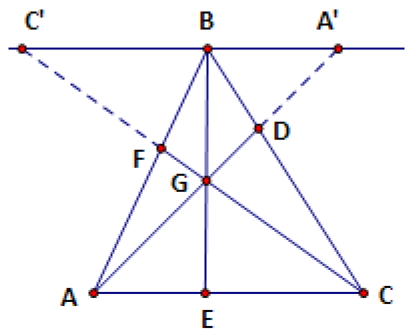
Proving Ceva's Theorem

The proof of Ceva's Theorem is done in two parts, since it is an "if and only if" statement. In your presentation, you will be proving Part 1. This part states that, using the diagram shown here, if the three cevians are concurrent, then $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$.



Follow this outline to complete the proof. Assume that you are given triangle ΔABC with cevians concurrent at G.

- Construct a line passing through B that is parallel to $\overline{AC}$.
- Extend segments $\overline{AD}$ and $\overline{CF}$ until they meet this line at points A' and C', respectively. You should have a diagram like the one shown on the next page.
- Using facts about parallel lines and similar triangle, you should be able to prove that there are several pairs of similar triangles in this diagram.



- Prove, using similar triangles, that each of the following equations is true:
  - $\frac{AE}{A'B} = \frac{EG}{GB}$
  - $\frac{CE}{BC'} = \frac{EG}{GB}$
  - $\frac{CD}{DB} = \frac{AC}{A'B}$
  - $\frac{AF}{FB} = \frac{AC}{BC'}$
- Combine these equations (using substitution and multiplication) to obtain the desired equation:

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

Using Ceva's Theorem

As an easy application of Ceva's Theorem, show why the medians of a triangle all intersect at a single point (called the "centroid"). The group doing Part 2 will prove the other direction of Ceva's Theorem and demonstrate some more complex applications.