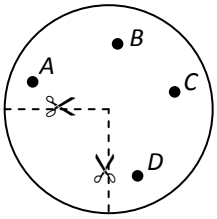
Lines on a Cone

Create a cone using the attached templates by cutting along the dotted lines and taping the cut edges together. The center of the circle that you are cutting out becomes the tip (or "vertex") of the cone.

270 Degrees

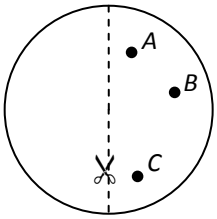
The "vertex angle" of a cone is the total number of degrees of a circle used to make the cone. So the cone shown below would be called a "270 degree cone." Cut along the dotted line, and tape the cut edges together.



- Once the cone is taped together, think about how we could draw "straight" lines between each pair of points. How can we draw the line from *A* to *D*?
- Describe in your own words what straight lines look like on a 270 degree cone. Are they really "straight"?

180 Degrees

Now create a 180 degree cone as shown below.



- Is there more than one way to connect *A* and *C* with a straight line? Does this violate one of Euclid's axioms?
- How can we define the idea of distance on a 180 degree cone? Using your definition, find the distances between each pair of points.

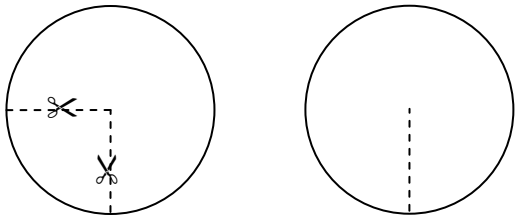
90 Degrees

Now create a 90 degree cone, and plot two points near the tip of the cone. Euclid says that a line can be extended indefinitely. Even though our cone is finite, we could imagine that it extends forever away from the tip, so we could theoretically extend lines forever.

- Draw a line connecting your pair of points and extend it as far as possible. You may need to un-tape and re-tape your cone multiple times.
- You may have noticed that your line intersects itself. Does this violate one of Euclid's axioms?
- Does every line on a 90 degree cone intersect itself if you extend it far enough?

450 Degrees!?

How is it possible to have a cone with an angle greater than 360 degrees?



Cut a slit in a circular disk, and insert a 90 degree section, taping the two sides of the section to the two sides of the slit. You have created a 450 degree cone!

- Place some points on your cone, and explore the properties of straight lines on this strange surface. Use the ideas of un-taping the cone and cutting extra slits to make drawing lines easier.
- Which of Euclid's line axioms are true on the 450 degree cone? Is the parallel postulate true?