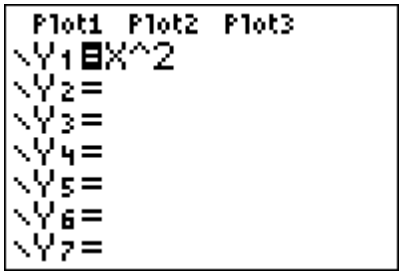
Section 1.5: Zooming to Estimate Derivatives

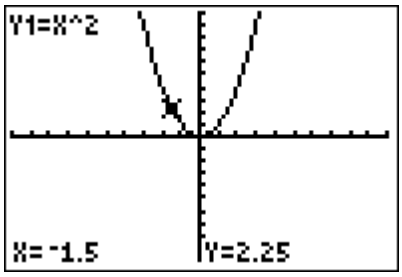
When we zoom in at a point on most well-behaved functions, eventually the graph looks like a line. If we can estimate the slope of that line, we will have estimated the derivative at that point.

Example: Let $f(x) = x^2$. Use zooming on your calculator to estimate $f'(-1.5)$.

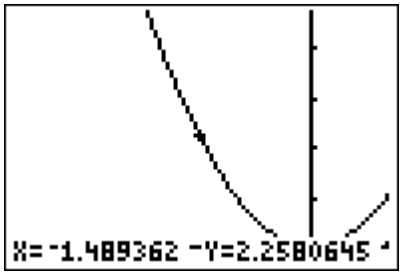
Step 1: Enter the function in your calculator and choose an appropriate viewing window. This window should include the point you are going to be zooming in on – in this case, the point is $(-1.5,\ 2.25)$. Press the [Graph] button to view your function.



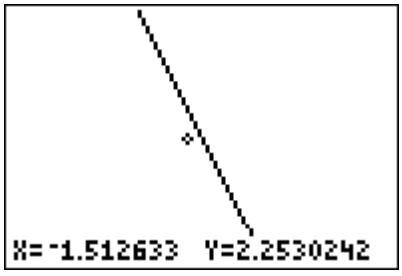
Step 2: Press the [Trace] button. While tracing, type in your x-value – in this case, type in -1.5. Press [Enter], and notice that the trace cursor moves to the point you specified.



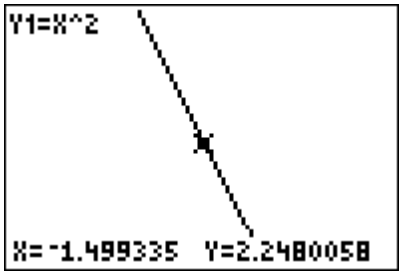
Step 3: Press the [Zoom] button and choose Option 2 ("Zoom In"). Your calculator will return to *approximately* the same location on your graph. Press [Enter] to zoom in.



Step 4: Continue pressing [Enter] to zoom in until the graph looks like a line. You may have to adjust the position of the zoom cursor to stay near the initial point.



Step 5: Press [Trace] and find a point as close as possible to the original point. Notice that the zoom cursor is not actually a point on the curve. Our goal is to find a point *on the curve* that is near the original point. *Do not* use the x- and y-values of the zoom cursor to estimate your slope. **This is the #1 mistake students make on these problems!**



Step 6: Use the point you found to estimate the slope. In this example, the point we find from our calculator is $(-1.499335, 2.2480058)$. Find the slope of the line connecting this point to our original point $(-1.5, 2.25)$:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.25 - 2.2480058}{-1.5 - (-1.499335)} \approx -2.9988$$

So we conclude that $f'(-1.5) \approx -2.9988$. If we had zoomed in farther, we would have found a point closer to our original point, and our estimate would be been more accurate.

Exercises

1. Zoom in farther to get a better estimate of $f'(-1.5)$.

2. Let $g(x) = x^2 - 3x + 10$. Use zooming on your calculator to estimate $g'(4)$.

3. Let $k(x) = \ln(x)$. Use zooming on your calculator to estimate $k'\left(\sqrt{2}\right)$.

Answers

1. $f'(-1.5) \approx -3$
2. $g'(4) \approx 5$
3. $k'\left(\sqrt{2}\right) \approx 0.707107$