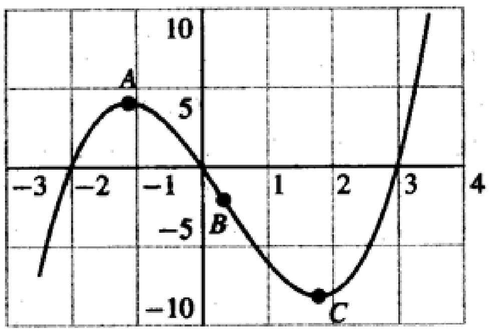
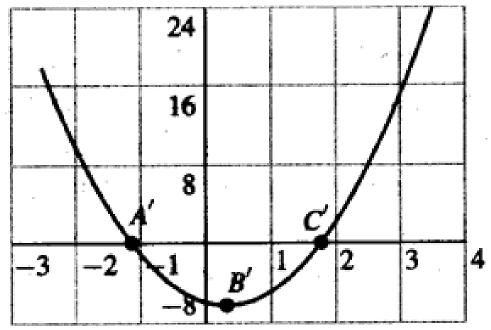
Section 1.6: Comparing a Function to its Derivative

The graphs of a function f and its derivative f' are shown below. Corresponding points (for example A and A') have the same x-coordinate.



(a) $y = f(x)$



(b) $y = f'(x)$

- The graph of f' describes **slopes** on the graph of f. For example, at B, the f-graph seems to have a slope of approximately -6, and so the f'-graph has a y-value of -6 at the point B'.
- Consider the following locations on the two graphs:

Graph of f
f is increasing to the left of A
f is decreasing between A and C
f is increasing to the right of C
f changes direction at A and at C

Graph of f'
f' is positive to the left of A'
f' is negative between A' and C'
f' is positive to the right of C'
f' changes sign at A' and at C'

- At the point $A(-1.1, 4)$, the function f has a **local maximum**. The input value $x \approx -1.1$ is called a **local maximum point** of f, and the corresponding output $f(-1.1) \approx 4$ is called a **local maximum value** of f. Similarly, f has a **local minimum** at point C, and $x \approx 1.8$ is the **local minimum point**, while $f(1.8) \approx -8$ is the **local minimum value**. At these points, the slope of the function f is equal to zero, and note that f' has roots at A' and C'.
- Points where the slope of a function equals zero are also called **stationary points**. The points A and C are stationary points for f, while B' is a stationary point for f'.
- At the point B, the concavity of the function f changes from **concave down** to **concave up**. We say that the x-coordinate of B (or sometimes B itself) is an **inflection point** of f. This is the point where the graph of f points most steeply downward, and note that f' has a local minimum at B'.