Abstracts

Plenary Speaker

Epidemiological Models in Action: from SARS to Influenza

Carlos Castillo-Chavez

Mathematical, Computational and Modeling Sciences Center, Arizona State University

The concept of threshold or tipping point, a mathematical expression that characterizes the conditions needed for a drastic transition between epidemiological states, is critical to the study of the transmission dynamics and control of diseases. The modeling and mathematical work of Sir Ronald Ross Ross1911 and his students Kermack and McKendrick (1927, 1932) established the field of mathematical epidemiology, intimately connected to the study of the dynamics of ecological systems, that include host-parasite systems. In their 1927 classical paper, A Contribution to the Mathematical Theory of Epidemics, W. O. Kermack and A. G. McKendrick introduced a simple deterministic model that turned out to capture rather well the qualitative dynamic behavior of single epidemic outbreaks. In this lecture, I will briefly review the history of the field of mathematical epidemiology in the context of communicable diseases. The emergence SARS in 2003 and the re-emergence of influenza in 2009 have not only highlighted the limitations of the existing mathematical framework and theory but pointed into natural directions for extensions. In this lecture, we introduce a generalized version of the Kermack-McKendrick framework that can be used to quantify the impact of variable waiting time distributions, infectious and quarantine, that is, delays disease dynamics. Memory-less distributions, geometric or exponential, set the baseline or null epidemiological scenarios used to test the relevance of delays on the dynamics of single epidemic outbreaks. We explicitly highlight final size relationships involving the control reproduction number, a function of transmission parameters and the means of distributions used to model disease or intervention control measures. Model results and simulations are used to highlight the kind of inconsistencies in forecasting that emerge from the use of specific parametric distributions*. The use of optimal control theory to assess the relative cost of intervention measures is briefly addressed. The work presented involves (*Hernandez Ceron, Nancy, Z. Feng) and the contributions of several collaborators including Sunmi Lee, Paula Gonzalez Parra, Leticia Velazquez and Carlos Torre.

Career Panel Discussion: Finding a Job

Panelists:

- Elizabeth Ayers (American Institutes for Research)
- Chandra Erdman (U.S. Census Bureau)
- Anthony Kearsley (National Institute of Standards and Technology)
- David Murillo (American Express)
- Barry Thorton (National Security Agency)

Student Paper Presenters

Models for Osmosis between the Cornea and Tear Film

Jennifer Bruhns (jlbruhns@yahoo.com)
University of Delaware [Advisor: Richard Braun]

The ocular tear film covers the exposed eye surface after each blink; it acts as a protective barrier as well as part of the optical system. Thinning and break up of the tear film can contribute to dry eye, which is a collection of symptoms many people suffer. The evaporation of the tear film causes the saltiness, measured by osmolarity (the concentration of ions) to increase. Transport of water via osmosis between the epithelial layers of the eye and the ocular tear film is driven by relative osmolarity differences between the tear film and each of the epithelial layers. A seven layer human epithelium model, based on of a mouse epithelium model by Levin and Verkman, is combined with a tear film model. Our model uses a system of ordinary differential equations to conserve mass and govern the transport of water throughout the system. We vary parameters such as permeability, layer thicknesses, evaporation rate and the time between blinks to study the dynamics of the tear film thickness, epithelial cell thickness and the osmolarity in each layer.

This work was done in collaboration with Richard Braun and Douglas Freeman.

Finite Elements Method for numerical solution of Maxwell's Equations in two dimensional Nanoplasmonics

Carmen Caiseda (ccaiseda@masonlive.gmu.edu)
George Mason University [Advisor: Igor Griva]

Solving Maxwell's equations for the electro-magnetic field in the presence of metal/dielectric interface allows the modeling and study of plasmonic effects in nanosystems. A 2D Finite Element Method is implemented to find the numerical solution to Maxwell's equation when a silver nanoparticle is present in the domain. The weak formulation of the derived PDE, absorbing boundary conditions, finite difference transformations from 1D Magnetic field to 2D electric field, and visualizations are derived and developed using Matlab.

Mathematical Modeling and Analysis of a Non-linear Large Deformation Plate Model with Applications to Micro-Air Vehicles

James Cameron (jcameroa@masonlive.gmu.edu)
George Mason University [Advisor: Padmanabhan Seshaiyer]

In this work we consider the development of a computational methodology to study stability and nonlinear dynamics of large deformation plate models. The main application will be the computational modeling of flexible wing designs for Micro Air Vehicles. Using a geometrically nonlinear Green strain-displacement formulation, a materially linear constitutive stress-strain formulation, and a Hamiltonian energy approach, wedevelop the governing differential equations for the axial and transverse displacements of the plate. We also develop an appropriate energy norm for a class of boundary conditionsand prove a stability estimate. The model developed will also be numerically validated for benchmark applications.

This work was done in collaboration with Charles Daly.

Multilevel optimization framework MG/OPT: applications and recent developments

Zichao Di (zdi@gmu.edu)
George Mason University [Advisors: Maria Emelianenko, Stephen Nash]

This talk will focus on recent progress related to the theory and applications of the multilevel optimization approach MG/OPT. The intent of MG/OPT is to use calculations on coarser levels to accelerate the progress of the optimization on the finest level. Uniform convergence of MG/OPT with respect to the problem size in an unconstrained setting is shown for a set of two-dimensional vector quantization problems with certain configurational restrictions. Another problem to be addressed is the extension of the MG/OPT framework to constrained problems which presents many challenges if taken in its full generality. In this talk, we will present the results of numerical experiments for a set of constrained optimal control problems where the newly developed constrained MG/OPT yields significant speedup comparing to existing techniques.

A Reaction-Diffusion Model for Cell Populations in the Colonic Crypt

Brooks Emerick (emerick@math.udel.edu)
University of Delaware [Advisor: Gilberto Schleiniger]

The human colon contains microscopic invaginations called crypts that are responsible for continuous epithelial cell reproduction. Inside the crypt, stem cells accumulate near the base and migrate up the crypt walls while proliferating and differentiating, until finally exiting through the top of the crypt as completely differentiated cells. Once the crypt establishes a specific cell organization, it splits into two new crypts. In cancerous colons, we see an abundance of stem cells, which leads to abnormal crypt fission and eventually polyp-like adenoma. We wish to model the formation of a normal colon crypt using a reaction-diffusion model, then perturb the system to gain insight into the mechanisms for disorganization of the crypt tissue.

This work was done in collaboration with Gilberto Schleiniger and Bruce Boman.

EigenValues In Action

Isaac Harris (iharris@udel.edu)
University of Delaware [Advisor: John Pelesko]

In this presentation we will explore an application of the spectral theorem for matrices that arises in the linear approximate of the coupled-mode equations. The matrices studied come up in the modeling of frequency conversion in optical parametric devices.We were interested in specific spectral properties of a particular form of matrices that arise when studying photon generation along a fiber.

This work was done in collaboration with Shuchi Agrawal, New Jersey Institute of Technology, David A. Edwards, University of Delaware, Joseph D. Fehribach, Worcester Polytechnic Institute, John Gounley, Old Dominion University, Richard Moore, New Jersey Institute of Technology, Takeshi Takahashi, University of Massachusetts and Jacek Wrobel, New Jersey Institute of Technology.

A Linear Nonlocal Convection-diffusion Equation

Zhan Huang (zhuang@math.psu.edu)
Penn State University [Advisor: Qiang Du]

We proposed and investigate a one-dimensional linear nonlocal convection-diffusion equation with periodic, Dirichlet and Neumann boundary conditions. This equation involves integrals instead of spacial derivatives, which reduces to a conventional convection-diffusion equation in the sense of distribution. From a deterministic point of view, we derived its dispersion relation, maximum principle and conservation law, under appropriate boundary conditions. From a stochastic perspective, we show that the solution of the deterministic equation is simply the probability density function of a compound poisson process. Applying Monte Carlo simulations for nonlocal solutions, and finite difference scheme for the corresponding local solutions, we show the effects of the model parameters and boundary conditions, and that the nonlocal equations coincide with its local counterparts as the horizon vanishes.

This work was done in collaboration with Qiang Du.

Optimizing Trips Down The Rivern

Grant Innerst (gi7584@ship.edu)
Shippensburg University [Advisor: Benjamin Galluzzo]

This past February we worked to determine optimal park usage as addressed in COMAPs annual Mathematical Contest in Modeling (MCM).The question we were given was to optimize the number of boats that could go down a river in a certain time period while simultaneously minimizing the number of campsites in use. There were additional restrictions on the speed of the boats and the length of the river. In this talk we will discuss the numerous models we created in attempting to solve the problem and how our interpretation of the problem was critical to our determining a final answer.

This work was done in collaboration with Allen Koederitz and Andrew Campbell.

Effects of Time-Dependent Stimuli in a Competitive Neural Network Model of Perceptual Rivalry

Suren Jayasuriya (smj20@pitt.edu)
University of Pittsburgh [Advisor: Zachary Kilpatrick]

We analyze a competitive neural network model of perceptual rivalry with time-varying inputs. Time-dependent stimuli generate novel network oscillations where both, one, or neither populations are "ON" at any given time. When a single population receives an interrupted stimulus, the fundamental behavior found is the temporally driven population phase-locking to its stimulus. Other behaviors are analyzed as bifurcations from this forced oscillation using fast/slow timescale analysis. Time-varying the input of both populations, we partition parameter space into particular oscillations types including mixtures of fusion and sole population dominance. Our results provide testable predictions for future psychophysical experiments on perceptual rivalry.

This work was done in collaboration with Zachary Kilpatrick.

The Optimal Mortgage Payment Problem

Christopher Jones (csj3@pitt.edu)
University of Pittsburgh [Advisor: Xinfu Chen]

Typically, the problem of optimally paying off a mortgage is described as an optimal stopping problem where the stochastic variable is the market interest rate. In this talk, I will demonstrate how the mortgage payment problem can be modeled as a free boundary problem. I will discuss a hedging argument which has not been put forth in previous research. Existence and uniqueness of a solution to the free boundary problem will be outlined and numerical simulations will be presented.

This work was done in collaboration with Xinfu Chen.

Computational docking of molecular wires to the reaction center of Rhodobacter sphaeroides

Byong Kwon (bkwon1@masonlive.gmu.edu)
George Mason University [Advisor: Igor Griva]

Given the worldwide interest in renewable energy, scientists have been exploring the possibility of using bacterial photosynthetic reaction centers to build a new generation of highly efficient photovoltaic devices. To build such devices, molecular wires (MWs) that serve as good conductors to transport electrons from and to the reaction centers are needed. The MWs must dock at specific binding sites within the reaction centers.

We explore computational models of docking MWs to the reaction centers. Such models can help in proposing suitable MWs for photovoltaic devices. For our modeling, we use the reaction center of Rhodobacter sphaeroides, a purple photosynthetic bacteria.

Measles Vaccination Refusal and its Effects on the Community

John McKay (JMcKayPitt@gmail.com)
University of Pittsburgh [Advisor: Eunha Shim]

Vaccine refusal has become an increasingly prevalent phenomenon in developed countries such as the US and UK. In fact, measles vaccine is reported to be the most frequently refused vaccine by parents in the US, resulting in measles outbreaks various communities, including San Diego in 2008. We investigate the effects of measles vaccination refusal in terms of increased risk of an outbreak and the disease burden. By using an agent-based model built with C++, we simulated the impact of measles vaccine refusal in Alleghany County, Pennsylvania. By varying the levels of the overall vaccine coverage of the children whose parents are skeptical about vaccination, we examined the risk of an outbreak, the age distribution of the infected individuals, as well as the probability of infection among the vaccinated ones who failed to develop immunity.

On the quasistatic approximation in the Stokes-Darcy model of groundwater-surface water flows

Marina Moraiti (mam328.pitt.edu)
University of Pittsburgh [Advisor: William Layton]

In this work we study the validity of the quasistatic approximation in the fully evolutionary Stokes-Darcy problem for the coupling of groundwater and surface water flows, as well as dependence of the problem upon the specific storage parameter, So. The quasistatic problem is obtained by setting the specific storage parameter equal to zero in the Stokes-Darcy equations. We prove that the solution of the Stokes-Darcy problem approaches the quasistatic solution as So approaches zero. We also estimate the rate of convergence.

Dynamics and Spectral Properties of Neural Network Structures

Jeff Moulton (jtm59@pitt.edu)
University of Pittsburgh [Advisor: Jonathan E. Rubin]

We consider a model neural network in which the activation of each neuron depends on inputs from other neurons. To encode network structure, we construct synaptic weight matrices, where elements represent the strength of the synaptic connections between neurons. Dynamics are given by n coupled first order differential equations relating the electrical activity of each neuron to the synaptic efficacies and activity of connected neurons. We investigate links between weight matrix properties and dynamics. Differences in spectra for small world, nearest neighbor, and random networks affect stability of dynamics and Lyapunov dimensions.

Understanding parameter variability in Models of Influenza A Virus Infection

Elizabeth O'Reilly (ewo2@pitt.edu)
University of Pittsburgh [Advisor: Sarah Lukens]

Influenza A virus (IAV) causes a highly contagious, acute respiratory disease. Deterministic models of virus/host interactions provide estimates of biological parameters describing viral kinetics, but variability exists between different strains, individual patients, as well as uncertainty in data. We fit data from two different experimental studies to a model of viral dynamics, employing an ensemble modeling approach. Ensembles minimizing a cost function measuring model fit to data and corresponding parameter distributions are generated using an MCMC algorithm with parallel tempering. We use statistical analysis to better understand the differences between the strains and the patients, and find that there is significant variability.

Use of the string method to find minimal energy paths and energy barriers of droplets on superhydrophobic surfaces

Kellen Petersen (kellen@cimsnyu.edu)
New York Univesity [Advisor: Weiqing Ren]

Interest in superhydrophobic surfaces has increased due to a number of interesting advances in science and engineering. Here we use a diffuse interface model for droplets on topographically and chemically patterned surfaces in the regime where gravity is negligible. We then apply the constrained string method to examine the transition of droplets between different metastable/stable states. The string method finds the minimal energy paths (MEPs) which correspond to the most probable transition pathways between the metastable/stable states in the configuration space. In the case of a hydrophobic surface with posts of variable height and separation, we determine the MEP corresponding to the transition between the Cassie-Baxter and Wenzel states. Additionally, we realize critical droplet morphologies along the MEP associated with saddle points of the free-energy potential and the energy barrier of the free energy. We analyze and compare the MEPs and free-energy barriers for a variety of surface geometries, droplets sizes, and static contact angles ranging from partial wetting to complete wetting. We also introduce an unbiased double well potential in the diffuse interace model by introducing a chemical potential that is fixed for a given simulation. We find that the energy barrier shifts toward the Wenzel state along the MEP as the height of the pillars increases in the topographically patterned case while a shorter energy barrier exists and is more centered along the MEP for pillars of shorter height. More importantly, we demonstrate the string method as a useful tool in the study of droplets on superhydrophobic surfaces by presenting a numerical study that finds MEPs in configuration space, critical droplet morphologies and free-energy barriers which in turn give us a greater understanding of the free-energy landscape.

This work was done in collaboration with Weiqing Ren.

A Comparative Analysis of the Efficiency of Multi-trials of the Warner and Simmons' Randomized Response Techniques

Brinita Ricks (brinita.ricks@gmail.com)
Wilson College [Advisor: Karen Adams]

This research reports upon the efficiency of multi-trial Warner and Simmons Randomized Response Techniques (RRT). RRT is used when attempting to solicit direct responses to sensitive question. In this research, participants were directed to respond to a sensitive question in three ways; the use of a single trial of the direct method; and the use of three trials of both the Warner and Simmons method. Results indicate that responses obtained using randomizers are better estimates of the actual population and that the multi trial Simmons model was proven to be superior to that of both the Direct and the multi trial Warner method.

Slow coupling in paired neurons

Jonathan J. Rubin (jjr37@pitt.edu)
University of Pittsburgh [Advisor: Jonathan E. Rubin]

We model the interactions of two identical conductance-based neurons with a system of differential equations. These neurons are coupled through shared external potassium and exhibit periodic synchronized bursting patterns but anti-phase spiking patterns within these bursts. By interpreting voltage as a fast variable and potassium as a slow variable, and recognizing that temporal variations in potassium constitute a form of weak interactions superimposed on the shared baseline external potassium level, we use weak coupling to analyze the desynchronization of spikes despite virtual synchrony during the quiescent portion of neurons periods.

This work was done in collaboration with Jonathan E. Rubin, Bard Ermentrout.

Modeling scratch rhythms in turtle

Abigail Snyder (acs73@pitt.edu)
University of Pittsburgh [Advisor: Jonathan E. Rubin]

A central pattern generator (CPG) is a population of neurons producing rhythmic or repetitive behavior (i.e. scratching, walking, masticating) without requiring rhythmic input to the population. Turtles are observed to produce several rhythmic motor patterns in response to stimuli, in particular rostral scratch, pocket scratch, caudal scratch, and forward swim We implement a proposed CPG for rostral scratch and caudal scratch as a system of relaxation oscillators and discuss preliminary results.

Clustering Genes in HL-60 Leukemia Cells by Expression Patterns and Genetic Promoters to Understand Changes in Gene Expression

Natalie Stanley (Stanleyn@dickinson.edu)
Dickinson College [Advisors: Jeffrey Forrester, Michael Roberts]

The HL-60 cell line can be used as an experimental model for acute myeloid leukemia, an important hematological cancer. These cells can be induced to behave normally in culture by a variety of different compounds. As the cancerous cell changes into one that is macrophage-like, its gene expression patterns are altered. To analyze these changes, we performed a cluster analysis of microarray data and grouped the genes based on their temporal expression patterns throughout this transition. The control of gene expression is orchestrated by a class of proteins called transcription factors, whose binding sites are present in the promoters of the cellular genes. In the second-stage of our analysis, we are investigating whether a clustering of the genes based on their promoter structure can reproduce the groups derived from the expression pattern analysis. Several potential metrics and clustering heuristics for the promoter analysis will be presented, and the challenges working with data sets of this nature will be discussed.

This work was done in collaboration with Nicole Briceno, Jamie Bugel, Catherine Campbell, Mary Dickinson, Trevor McCarthy and Phoebe Oldach.

Effect of stretch-dependent proliferation on collective cell migration

Tracy L. Stepien (tls52@pitt.edu)
University of Pittsburgh [Advisor: David Swigon]

Collective cell migration plays an important role in maintaining the cohesion of epithelial cell layers and in wound healing. A recently developed mathematical model of cell layer migration based on an assumption of elastic deformation of the cell layer leads to a generalized Stefan problem. The model is here extended to incorporate stretch-dependent proliferation, and the resulting partial differential equation system is solved analytically and numerically. We find that a large class of constitutive equations for the dependence of proliferation on stretch leads to traveling wave solutions with constant wave speed.

An Analytical Approach to Green Oxidation

Diego Torrejon (dtorrejo@masonlive.gmu.edu)
George Mason University [Advisor: Maria Emelianenko]

In this talk, we study the problem of suicidal inactivation of oxidation catalysts. We formulate a system of differential equations that models chemical reactions and analyze its numerical and analytical properties. Noticing its similarity with Michaelis-Menten system, we have been able to develop quasi-state approximation that together with perturbation techniques has allowed us to derive an approximate solution. The main goal is to estimate the rates of the reactions for deeper understanding of the system.

Classifying Tear Film Lipid Layer Micrographs

Kaijing Wang (kevinwkj@udel.edu)
University of Delaware [Advisor: Richard Braun]

The goal of the Lipid Layer Structure Classification project is to classify thousands of grey-scale images of lipid structures, collected from human eye tear film lipid layer, into reasonable groups, or clusters. The lipid microscope images are gray scale images with pixel values that vary by mean, standard deviation and other statistical data, and differ in background with some dark and the others bright, and in shape of objects in the background that affects the amount of edges. So by collecting 7 measurements of statistical data per image (including mean, standard deviation, variance, skewness, kurtosis, Canny edge fraction and Hausdorff dimension), we were able to create a matrice of numbers representing the proximity of images to each other, which subsequently allowed images to be grouped by Ad-Hoc statistical approaches in MATLAB. The best classification that we found from this approach led to 9 clusters of images. And by adding and revising proper and suitable statistical data, we will be able to improve the clustering for better classification on larger sample space.

This work was done in collaboration with Richard Braun, Tobin Driscoll, Robert Paul, Ewen King-Smith, Jason Nichols and Kelly Nichols.

Conditions for Particular Spanning Trees in Graphs

Wiseley Wong (wwong@math.udel.edu)
University of Delaware [Advisor: Sebastian Cioaba]

We provide a sufficient eigenvalue condition for the existence of 2 or 3 edge-disjoint spanning trees in a regular graph. We exhibit a family of graphs that show our bounds are essentially best possible. A conjecture for more than 3 edge-disjoint spanning trees is also mentioned.

This work was done in collaboration with Sebastian Cioaba.

Student Poster Presenters

Dynamic Buckling of Elastic Beam

Qichuan Bai (bai@math.psu.edu)
Pennsylvania State University [Advisor: Andrew Belmonte & Qiang Du]

We consider a thin elastic beam subject to the sudden impact of projectile on the end, with the other end to fixed. We then get a coupled PDE system for normal (lateral) displacement u and tangential (compressional) displacement v , which is one hyperbolic equation plus a elliptic type problem in some parameter region. We have identified some interesting patterns during numerical simulation that have been observed in the lab such as post buckling. We would like to either give a more stringent criteria of the parameters, where we can compare the model with physical phenomenon. On the other hand, we would like to extend our model to contain the extreme case of breaking in the near future.

Modeling the Effect of Diversity in Host Plant-Herbivore-Predator Interactions

Karissa Smith (kasmith1@ursinus.edu)
Ursinus College [Advisor: Mohammed Yahdi]

Predator and plant diversity can control Potato leafhopper (PLH) pest damage to the host-plant Alfalfa. New models of systems of differential equations were constructed using age structures, Beverton-Holt, Allee effect, and the Shannon diversity Index. Recent data and results on enemies and diversity hypotheses in filed experiments were used to determine parameter ranges and validate the models. Parameters were adjusted to predict outcomes for scenarios not covered by field experiments and to examine their roles. This work provided a frame work for designing cost-effective and environmentally safe strategies to minimize alfalfa damage, and utilize enemies hypothesis and polyculture diversity.

This work was done in collaboration with Allison Bugenis, Mohammed Yahdi.

Application of Fibonacci Sequences and Prime Numbers in Cryptography: A Comparison Study

Chad Vanorsdale (CVANOR01@rams.shepherd.edu)
Shepherd University [Advisor: Weidong Liao]

The Fibonacci sequences are numbers defined by F(1)=1, F(2)=1 , and F(n)=F(n-1)+F(n-2). The first 5 terms would be 1, 1, 2, 3, and 5. Among many applications of Fibonacci sequences, recent study has shown that Fibonacci sequences may also find its application in cryptography and secure communication, an area in which prime numbers have played an important role. In this poster, we present our comparison study between Fibonacci Sequences and prime numbers, with focuses on their application in cryptography. We will also demonstrate our software developed to generate Fibonacci sequences and prime numbers.

This work was done in collaboration with Osman Guzide.

Optimal Control Theory for a VRE Model

Clinton Watton (cwatt756@live.kutztown.edu)
Kutztown Univeristy [Advisor: Mohammed Yahdi]

Antibiotic resistance VRE (Vancomycin Resistant Enterococci) is one of CDC's top health concerns. Optimal Control Theory is used to determine efficient and economically favorable strategies to prevent outbreaks and to control the emergence of VRE in intensive care units. A new colonization-Infection deterministic model is used with parameter ranges from actual ICU data. Key controls included combinations of the levels of special preventive care, healthcare workers' compliance rates, and health and economical costs. In particular, variable and lower levels of special preventive care are shown to best control VRE and prevent outbreaks.

This work was done in collaboration with Emily Dougherty, Michael Dunlea, Mohammed Yahdi.

Tuberculosis models with contact tracing system and fast and slow dynamics

Tian Xing (xingt1@mail.montclair.edu)
Montclair State University [Advisor: Baojun Song]

Models are incorporate with the transmission process of TB disease in two sub-populations of a social network, because TB disease is spread via casual contacts or through an epidemiologically active social network (cluster) that includes at least one actively infected individual. On top of that, contact tracing is a primary means of disease control for infectious diseases with low prevalence, especially for TB. A common approach is to focus on people who do not belong to clusters and most at risk of contracting TB and developing severe forms of TB after contact with infectious individual. Models are going to interpret the tracking ratio of treated people after being contact with the infectious ones besides evaluate the second production rate.