Properties of Rings

Keep this reference guide handy while you work on Chapter 3 exercises.

Ring

A ring is a nonempty set R equipped with two operations (usually written as addition and multiplication) that satisfy the following axioms.

1. If $a, b \in R$, then $a + b \in R$	**closure under addition**
2. $a + (b + c) = (a + b) + c$	**associativity for addition**
3. $a + b = b + a$	**commutativity for addition**
4. There is an element 0_R in R such that $a + 0_R = a = 0_R + a$ for all $a \in R$	**additive identity**
5. For each $a \in R$ the equation $a + x = 0_R$ has a solution in R	**additive inverse**
6. If $a, b \in R$, then $a \cdot b \in R$	**closure under multiplication**
7. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$	**associativity for multiplication**
8. $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$	**distributivity**

Commutative Ring

A commutative ring is a ring R that satisfies this property.

9. For all $a, b \in R$, $a \cdot b = b \cdot a$	**commutativity for multiplication**

Ring with Identity

A ring with identity is a ring R that satisfies this property.

10. There is an element 1_R in R such that $a \cdot 1_R = a = 1_R \cdot a$ for all $a \in R$	**multiplicative identity**

Integral Domain

An integral domain is a commutative ring R with identity $1_R \neq 0_R$ that satisfies this property.

11. If $a, b \in R$ with $a \cdot b = 0_R$, then $a = 0_R$ or $b = 0_R$	**property of zero**

Field

A field is a commutative ring R with identity $1_R \neq 0_R$ that satisfies this property.

12. For each $a \in R$ with $a \neq 0_R$, the equation $a \cdot x = 1_R$ has a solution in R	**multiplicative inverse**