Map Coloring:
From Checker Boards to Möbius Strips

1. Introduction

This activity is about coloring regions on different kinds of maps such that no two bordering regions are the same color.

2. Edge-to-edge line maps

Activity. Students are given a blank sheet of paper and told to draw 4 lines all the way across the paper (edge-to-edge) in any direction. The lines are now borders of regions, that is, each student has created a map. The students are now asked to color this map so that no two regions that share a border are the same color; although, two regions that touch only at a single point can be the same color. At first, students can use any number of colors, and then are asked to use the fewest colors possible.

They will find that the map can be colored with 2 colors (like a checker board). They are then asked to prove any such map can always be colored with 2 colors, no matter how few or how many lines are added as long as each line goes all the way across the page.

Facilitator Notes. The simplest proof is an induction on the number of lines, although the technical language of induction need not be used.

3. A map of the Western United States

Activity. Students are given a large uncolored map of the Western half of the continental United States and asked to color it using the fewest number of colors. Again, if two states share a border (except for a single point), they must be different colors.

After the students have figured out that this map can be colored using four colors, they are asked if it can done using three. If "Yes," then do it. If "No," explain why not.

Four-Color Theorem. Tell students about the four-color theorem, which states that any (planar) map can be colored with four or fewer colors.

Facilitator Notes. The proof that the US map cannot be colored with three or fewer colors is a parity argument.

4. A map on a Möbius strip

Activity. Students are given a Möbius strip with a map drawn on it, and asked to color it using the fewest number of colors. Again, if two regions share a border (except for a single point), they must be different colors.

After the students have figured out that this map can be colored using six colors, they are asked if it can be done using five. If "Yes," then do it. If "No," explain why not.

Facilitator Notes. The proof that it cannot be done with five or fewer colors relies on the fact that each of the six regions borders every other region.