Tips for solving problems
Shippensburg Area Math Circle
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Let's start with pointing out a difference between an exercise and a problem. An **exercise** is something we want to do, and we already know what it takes to do it. For example, if we are going to run a marathon, which is 26 miles, we know that we need to take steps. We're going to take over 40,000 steps and it will be difficult, but we know how to do it. A **problem** is something we want to do, and we don't yet know what it will take to do it. For example, if we are going to climb a mountain, we might not even know how to start. Which path should we take to reach the top? Do we need rope and other climbing gear? Should we bring water and food? Maybe we will get stuck halfway up, and need to climb down and start over again.

We are going to work on problems!

For many of the problems no one (including the adults!) will know what to do. But we can enjoy exploring and learning together if we all do the following:

1. Work hard.

2. Be willing to try things even if we don't know how to do them.

3. Rejoice in confusion.

4. Focus on exploring a problem, not solving it.

5. Ask questions.

6. Help those around us discover and learn too.

7. Be able to explain what we are doing and what we are thinking.

Ok, if we are going to work on problems and don't know what to do, then we need some tips for getting started, keeping at it, and making discoveries.
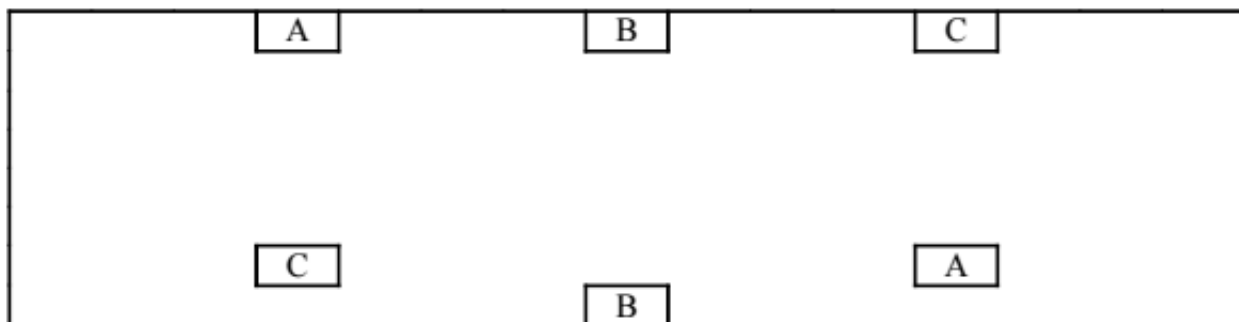

This document is based on material created by Joshua Zucker.

1. Change the problem

If we are having trouble with a problem, let's change it. We might learn something from solving one that is easier.

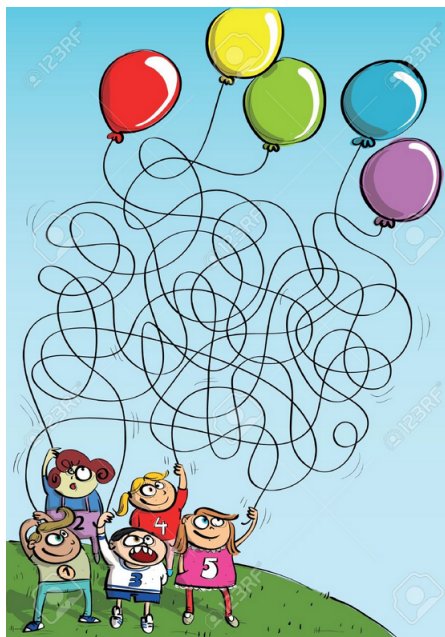1. Connect A to A, B to B, and C to C without crossing lines or leaving the box.

2. Find the sum of the digits in the product $555,555,555 \times 999,999,999$.

2. Work backwards

Sometimes we know how things should end, but we are not sure about the start or the steps in between.

3. Can you do the Problem-Solving Salute?

4. Which child is holding the red balloon on the left?

Problem #2 by Altha Rodin.

3. Organize work

Sometimes we need to consider lots of possibilities. It can really help to have an organized plan for considering them.

5. How many different ways can the four children Abbey, Bob, Cathy, and David line up for recess?

6. How many rectangles are there in a 6×6 chessboard?

4. Be patient

A problem worth thinking about can't be solved quickly, or we wouldn't be thinking about it. Even when we are clever and see a shortcut, there can still be work to do.

7. Arrange the numbers 1 through 6 into a "difference triangle," where each number in the row below is the difference of the two numbers above it. For example,

$$
\begin{array}{ccccc}
6 & & 4 & & 5 \\
& 2 & & 3 & \\
& & 1 & &
\end{array}
$$

almost works, but 3 is not $5 - 4$.

8. Find a 9-digit number, using each digit 1 through 9 once, such that the number made by the first n digits is divisible by n. For example, suppose we choose $786,941,532$. Then 2 is divisible by 1 and 32 is divisible by 2, but 532 is not divisible by 3.

5. Get your hands dirty

We can't expect to solve a problem if we don't do anything. We can always try some examples, guess and check, or change the problem. We can write down what we know, what we don't know, or what we would like to know. Never stare at a blank piece of paper. Once you get your hands dirty, you can produce some ideas. And if you're patient, too, you can write down a lot of stuff. This is when organizing your work can really help.

9. What is the smallest number that can't be written by subtracting a prime from a square? For example, $1 = 4 - 3$, $2 = 9 - 7$, and so on.

10. Around the Round Table sit the 64 knights. Sir Cumference then walks around and around the table, saying alternately, "go, stay, go, stay," for each knight he passes. As they obey and go, of course, Sir Cumference does not talk to the empty chairs. Which knight will be the last one remaining? [Remember, get your hands dirty. Try starting with fewer knights, draw some pictures, look for patterns.]