Answers.

(4) Fill in the missing steps of the proof for the following statement: An *oblong* number is a positive integer of the form $k(k+1)$ for some positive integer k. The sum of two successive oblong numbers is twice a perfect square between them.

Proof. Let n and m be successive oblong numbers. This means there is a(n) integer k such that $n = k(k+1)$ and $m = (k+1)(k+2)$, and so

$$\begin{aligned} n+m &= (k(k+1)) + ((k+1)(k+2)) \\ &= 2(k+1)^2 \end{aligned}$$

Notice that $(k+1)^2$ is a perfect square between n and m. □

(5) Consider the statement: If x and y are positive real numbers and $x \neq y$, then $x + y > 4xy/(x+y)$. This is a universal statement, and suppose you want to prove it by using particular arbitrary real numbers Write the first sentence of your proof.

Let x and y be positive real numbers such that $x \neq y$.

(6) Consider the following statement: For every positive integer n less than 15, $n^2 + n + 41$ is prime. Explain how you would prove this statement is true, and be sure to mention your method of proof.

I would evaluate n^2+n+41 for the numbers $1, 2, 3, \ldots, 39, 40$ and verify we get a prime number. This is the method of exhaustion.

(7) Consider the following statement: If n is a positive integer, then $n^2 + n + 41$ is prime. We want to show that this statement is false.

(a) Write the negation of this statement.

(b) Prove that the original statement is false by finding a counterexample.

(a) There exist an integer n such that n^2+n+41 is not prime.

(b) Consider $n=41$. Then we have $41^2+41+41 = 41(43)$, which is not prime.