

# The Pancake Problem\*

Shippensburg Area Math Circle

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The chef at the local cafe holds the record for making the quickest stack of pancakes in PA. However, in her rush to get the pancake orders out the door, she ignores how the stacks look as they leave the griddle.

No two pancakes are the same size, and the chef doesn't try to put them in order with the largest on the bottom of the stack.

The waiter delivering the pancakes tries to rearrange the stacks on his way out of the kitchen, but he is only able to make one sort of move in his quest to adjust the stack. He can stick the spatula somewhere in the stack and flip, in one motion, everything that sits above the spatula.

The illustration in Figure 1 shows an example of a legitimate move.

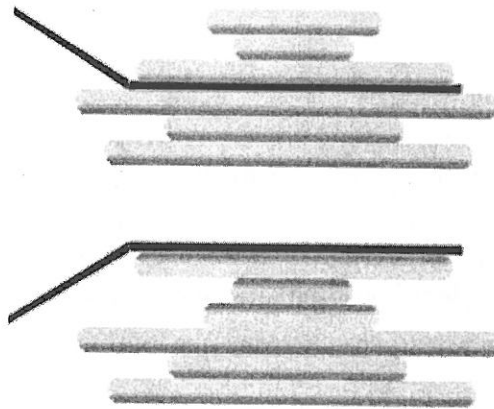


Figure 1: Pancake sorting operation. Image created by Derrick Coetzee, retrieved from Wikipedia: [https://commons.wikimedia.org/wiki/File:Pancake\\_sort\\_operation.png](https://commons.wikimedia.org/wiki/File:Pancake_sort_operation.png)

The waiter's goal is for the stack of pancakes to be increasing in size from the top of the stack to the bottom, so by the time it gets to the customer it looks like a pyramid of pancakes.

He notices that it is easier for some stacks than for others. For example, every once in awhile the chef happens to hand him a plate where the pancakes are already in the correct order, so he doesn't need to make any flips at all.

Other times it takes him several flips to get the pancakes arranged the right way.

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## Challenges

**Question 1.** How many flips are needed if the waiter is handed 0 pancakes? How many for 1 pancake? How many for 2 pancakes?

(Write down each stack and keep track of how many flips are needed for that stack.)

The waiter wants his shift to end at some point, so he would like to know the worst-case number of flips needed on a stack of 3 pancakes. He would also like to know the worst-case number of flips for 4 pancakes. These are called the **pancake numbers** for 3 and 4 pancakes.

It turns out that finding pancake numbers is a hard problem in general! The pancake numbers are all unknown for more than 20 pancakes.

We can calculate some small cases by hand though.

Always try to use the most efficient flipping strategy you can find.

**Question 2.** How many different stacks of 3 pancakes are there? (Write them down!)

**Question 3.** What is the worst-case number of flips required to rearrange 3 pancakes?

**Question 4.** How many different stacks of 4 pancakes are there?

**Question 5.** What is the worst-case number of flips required to rearrange 4 pancakes?

## Exploration

**Question 6.** Can you come up with a general set of steps for rearranging a stack of pancakes that would always work? How many steps would it take?

A set of steps like in the question above would give us a ceiling for the pancake numbers. The waiter would be happy to hear this news, because it means his shift will end even if someone orders a large stack of pancakes!

**Question 7.** What do the worst stacks look like? Can you come up with a stack of 5 pancakes that takes at least 5 flips?

## More reading

This problem was first written down by Jacob Goodman under the pen name “Harry Dweighter”, in [1].

1. Dweighter, Harry (1975), “Elementary Problem E2569”, *American Mathematical Monthly* 82: 1010.
2. Simon Singh, “Flipping Pancakes with Mathematics,” *The Guardian*, Nov. 14, 2013