# Chapter 12

## A Cheerful Fact

## The Pythagorean Theorem

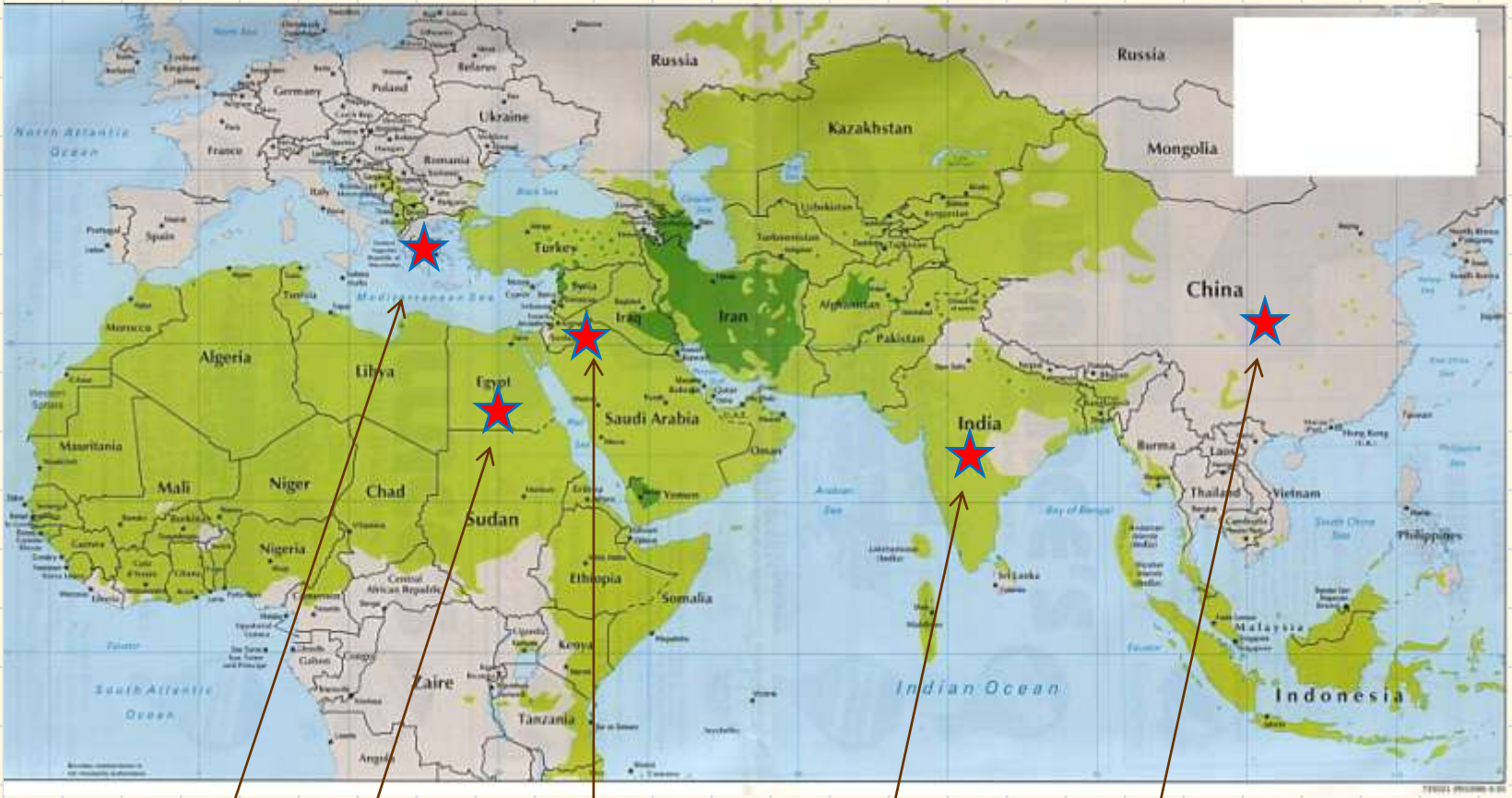
# Outline

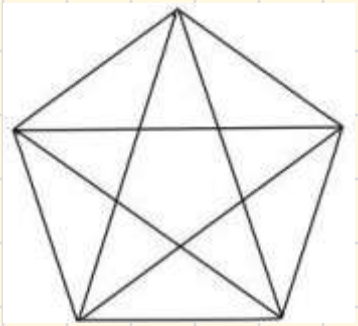
- Brief History
- Map
- Pythagoreans
- Algebraic Square Proof
- Geometric Square Proof
- Proof without Words
- More Proofs
- Euclid's *Elements*
- Triples
- Coordinate Geometry
- Time Line

# Brief History

- Origins are hard to trace
- Greek tradition associated the theorem with Pythagoras (5th century B.C.)
- Evidence that cultures know the theorem, are all over the world
  - Mesopotamia, Egypt, India, China, Greece
- Oldest references are from India dating from the 1st century B.C.
  - Artifacts reads that the diagonal of a rectangle "produces as much as is produced individually by the two sides."
- References of triples of whole numbers that "work" as sides of right triangles.
  - (3,4,5); (119, 120, 169)
- Evidence suggests that the Pythagorean Theorem was known by all mathematical cultures well before the time of Pythagoras himself

# Map of Locations



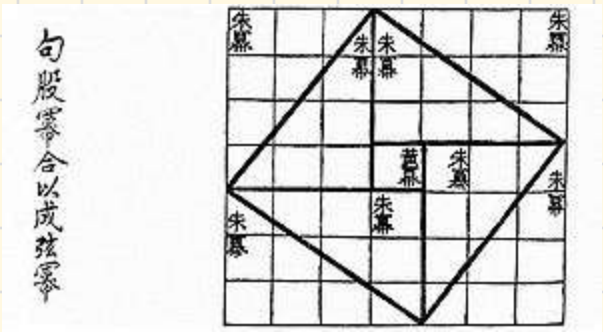


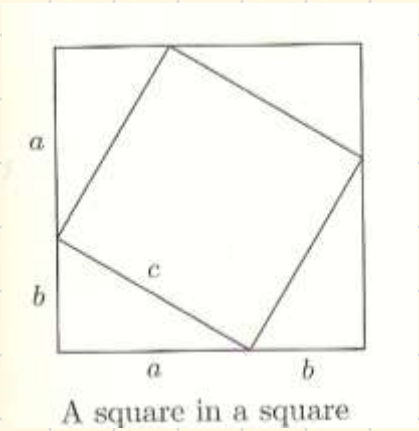
# Pythagoreans

- Pythagoras known mostly by the work of his disciples
- Choose his disciples simply by looking at them
- Eat very little–no meat or beans
- Must erase body impression from bed sheets to avert the “evil eye” (negative power)
- Symbol was a pentagram with a star in it
- Not allowed to:
  - Wear rings
  - Stir fire with iron
  - Speak of Pythagorean matters in the dark

# Algebraic Square Proof

- Chinese sources have earliest proofs
  - Square in Square
- Arranges 4 identical triangles around a square whose side is their hypotenuse
- Since all triangles are identical, the inner quadrilateral is a square of side c
- Big square has sides (a+b), area : **$(a+b)^2 = a^2 + b^2 + 2ab$**
- Decomposes into a square with area **$c^2$** and 4 triangles with area ½ ab
- So $c^2 + ½\ ab = a^2 + b^2 + 2ab$ → **$a^2 + b^2 = c^2$**

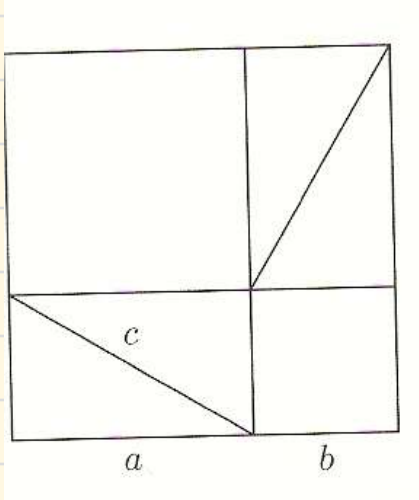




A square in a square

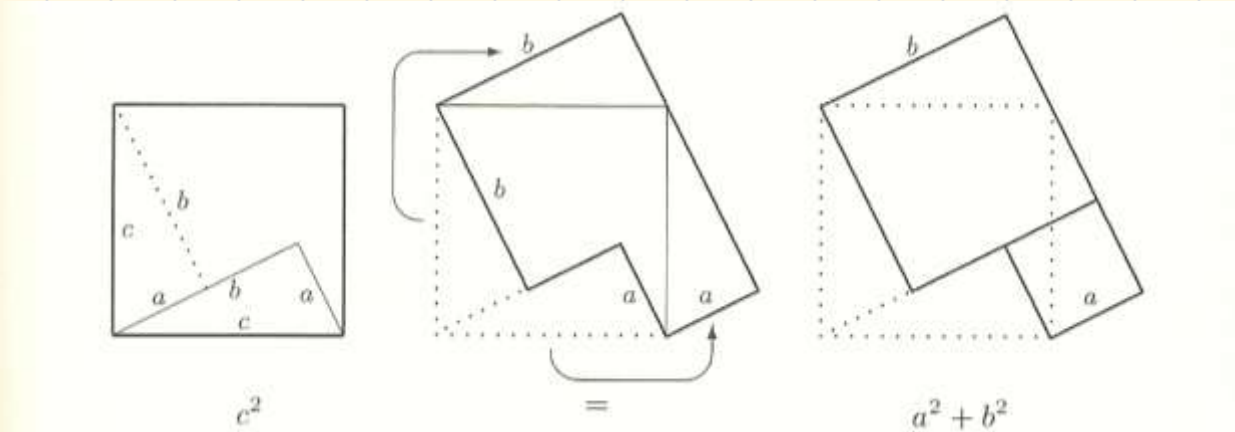
# Geometric Square Proof

- Get the same square of side (a+b), but the four triangles have been moved into rectangles
- $a^2 + b^2 = c^2$



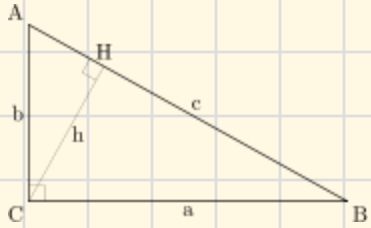
# Proof without Words

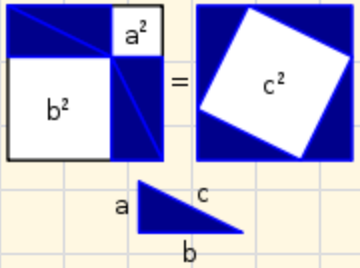
- 9th century Islamic mathematician of Baghdad created a proof without words

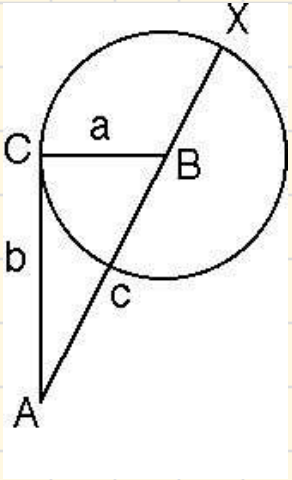


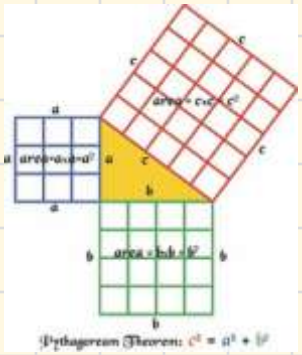
# More Proofs

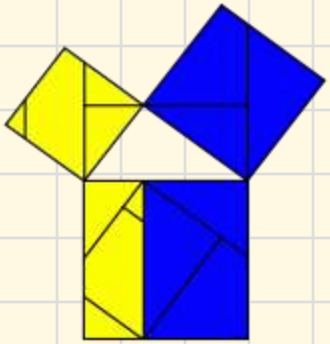
- There are many ways to prove the Pythagorean Theorem
- There are books devoted solely on proofs of the Pythagorean Theorem

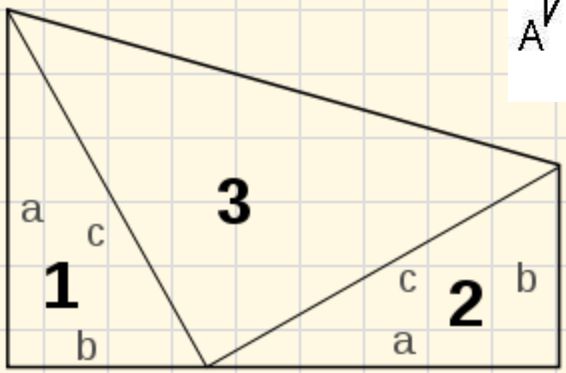


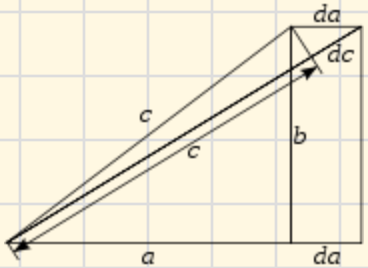



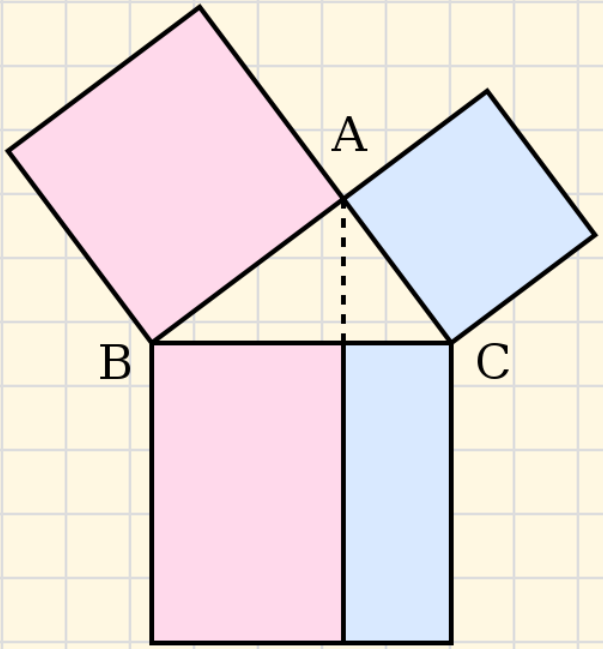




# Euclid's *Elements*

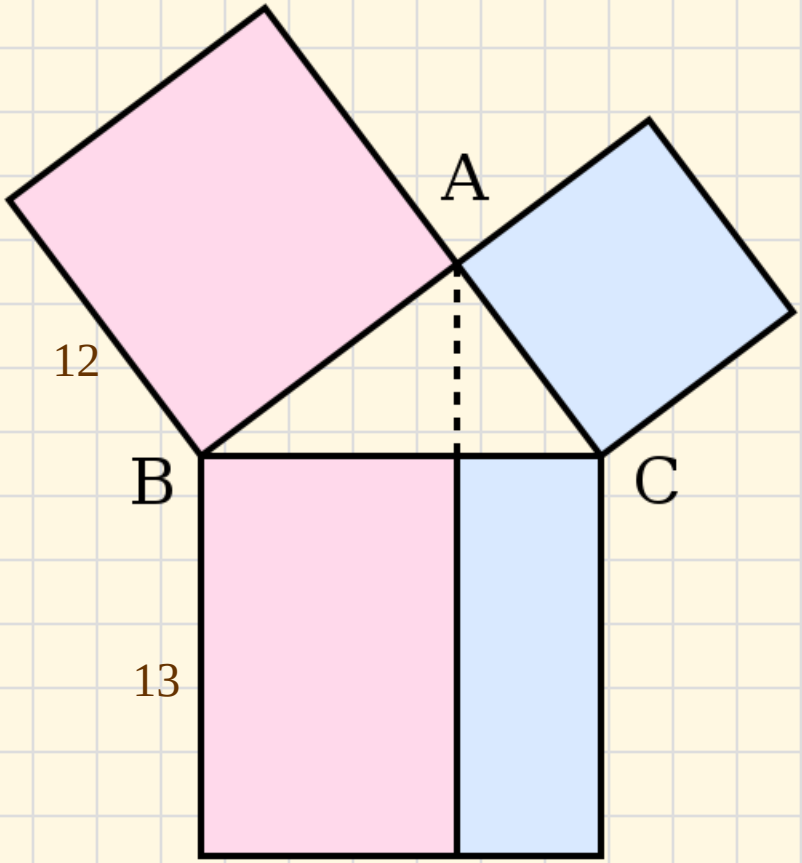
- Most famous is in the 1st book of Euclid's *Elements*
  - *47th Postulate:* In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.




- Drops a perpendicular for the upper vertex of the right triangle, splitting the bottom square into two pieces
- Using facts about triangles and parallelograms, he proves that each piece of the bottom square is equal to the corresponding smaller square. (He shows how to divide the big square into two pieces whose areas match the areas of the two smaller squares

# You try!

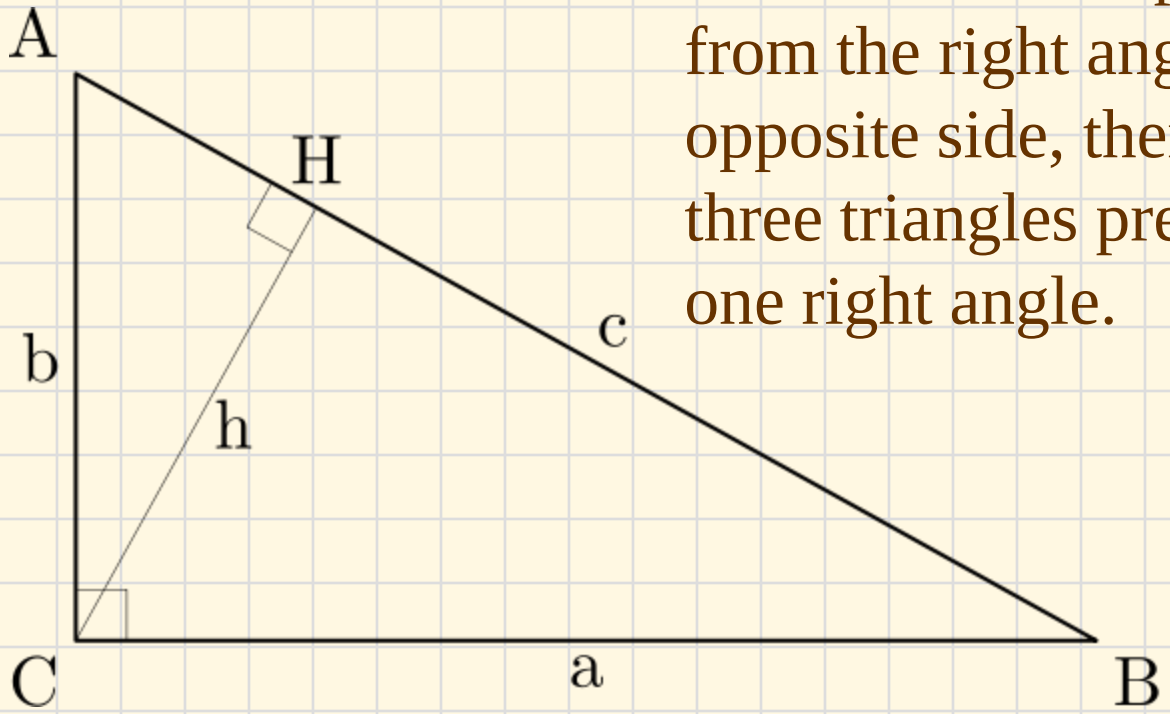
- By using Euclid's 47th postulate and the information given, find the area of the blue square which will also equal the area of the blue rectangle



# Euclid Cont.

- A newer proof of the theorem was found not too long ago.

-If one starts with a right triangle and draws a line perpendicular from the right angle to the opposite side, there are now three triangles present all with one right angle.



# Pythagorean Triples

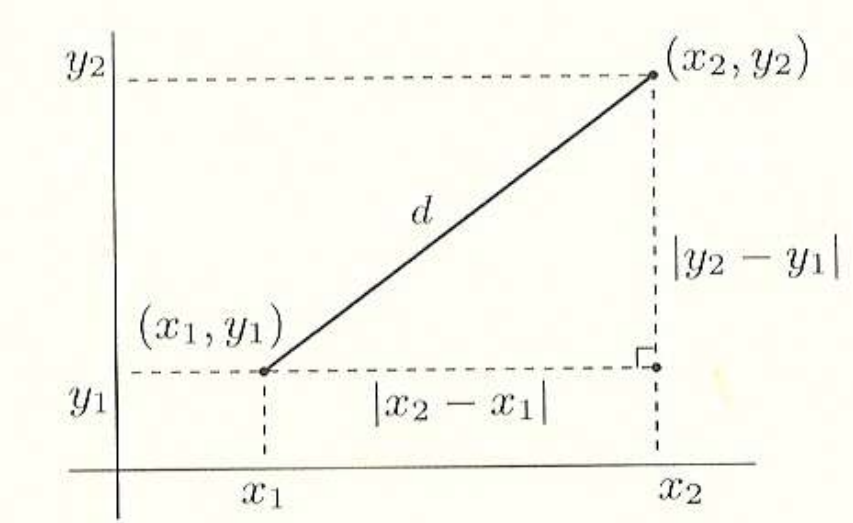
- Triples were used to make square corners and to find lengths
- List of triples to 100:
  - (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (13, 84, 85), (16, 63, 65), (20, 21, 29), (28, 45, 53), (33, 56, 65), (36, 77, 85), (39, 80, 89), (48, 55, 73), (65, 72, 97)

# The Theorem in Coordinate Geometry

- The distance between two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- If this were done on the surface of a sphere it would not work.



# Time Line

- 2500BC in Egypt first Pythagorean triples discovered
- 1750BC Mesopotamia-more triples
- 8th-2nd century BC-”Baudhayana Sulba Sutra” in India
- 569-475BC-Pythagoras
- 300BC Euclid

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