

# FERMAT'S LAST THEOREM

**Pierre de Fermat**

1601 –1665

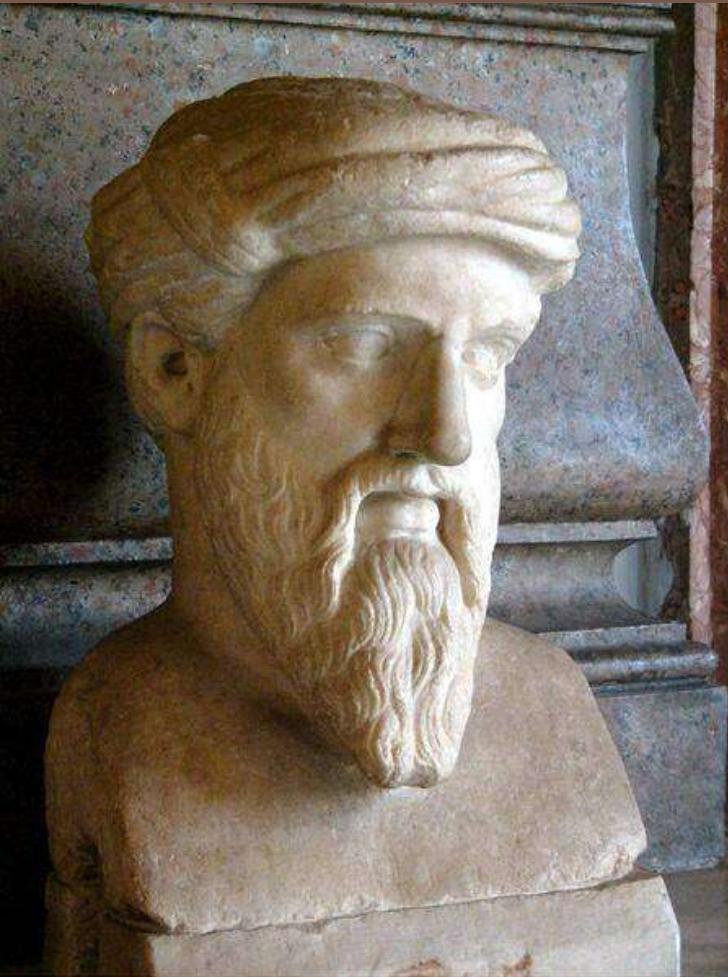


By  
Julie Kreps & Dustin Riden

# Timeline

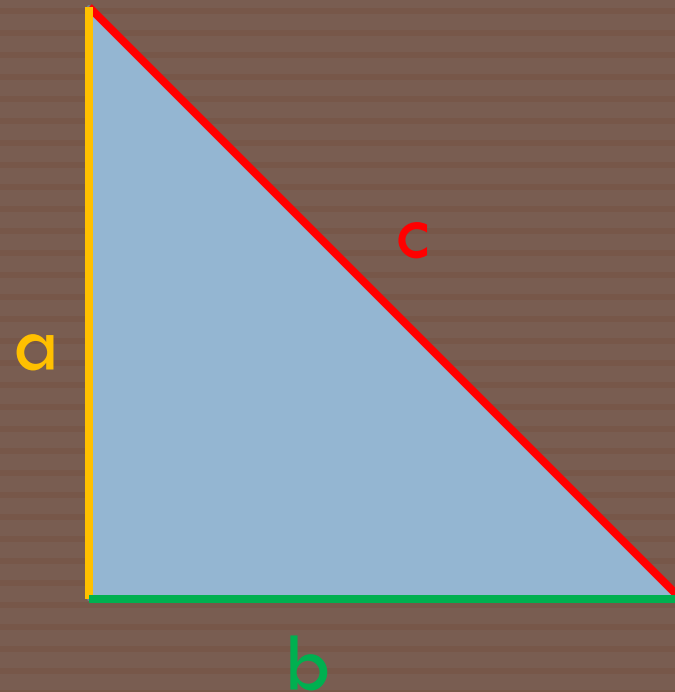
- 500 B.C – Pythagoras
- 200 A.D. – Diophantus
- 1601 – Pierre de Fermat was born.
- 1665 – Pierre de Fermat died.
- After 1670 – Fermat’s son Samuel republished the work of Diophantus with some of his father’s notes included. The only theorem Fermat left unproven was that  $x^n + y^n = z^n$  has no whole number solution for any power except  $n=2$  . Proofs were available for  $n=3$  and  $n=4$ , but not for  $n=5$  or greater
- 1808 – Sophie Germain published work where she split the proof into two cases and showed that there are no solutions in which none of  $x$ ,  $y$ , or  $z$  is divisible by the exponent  $n$  if the prime exponent  $n$  satisfied the additional constraint that  $2n+1$  should also be prime. Fermat’s theorem was now proved for up to 100 in the first case of Germain’s proof.
- 1825 – Lejeune Dirichlet gave a partial proof of the second case of  $n=5$  to the Paris Academy.
- After 1825 – Adrien-Marie Legendre finished Dirichlet’s part of the proof of the second case  $n=5$  .
- Late 1820’s – Gabriel Lamé proved the theorem for  $n=7$  .
- 1830 – Lamé used a complex number to factor  $x^n + y^n$  and proposed a draft of the full theorem to the Paris Academy. The proof did not work, but gained the interest of Ernst Kummer.
- After 1830 – Ernst Kummer gave a proof for any exponent that is a “regular” prime, but the proof did not work for other primes.
- 1909 – Paul Wolfskehl established a cash prize for anyone who could find a successful proof.
- 1993 – Andrew Wiles announces a proof, but it is incomplete.
- 1994 – Andrew Wiles completes the proof.

# Pythagoras (570BC-495 BC)



- Pythagorean triples

- $a^2 + b^2 = c^2$



# Using Pythagorean theorem

$$1^2 + 2^2 = c^2$$

$$7^2 + 24^2 = c^2$$

$$50^2 + 120^2 = c^2$$

$$8^2 + 15^2 = c^2$$

$$9^2 + 40^2 = c^2$$

$$12^2 + 35^2 = c^2$$

$$16^2 + 63 = c^2$$

$$20^2 + 21^2 = c^2$$

# Using Pythagorean theorem

$$1^2 + 2^2 = 3^2$$

$$7^2 + 24^2 = 25^2$$

$$50^2 + 120^2 = 130^2$$

$$8^2 + 15^2 = 17^2$$

$$9^2 + 40^2 = 41^2$$

$$12^2 + 35^2 = 37^2$$

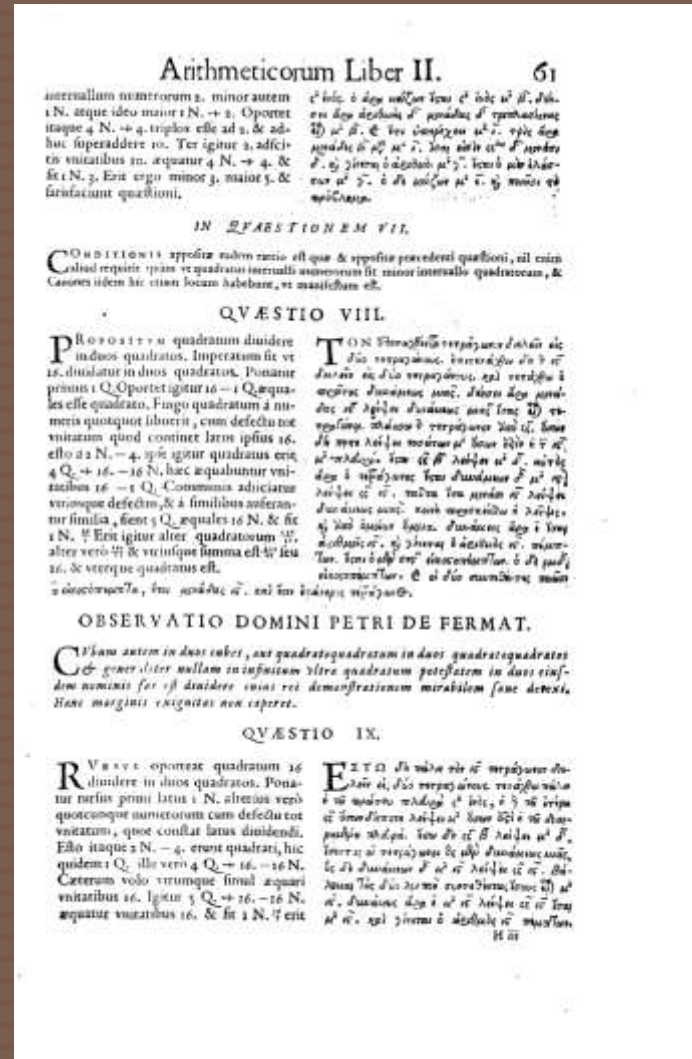
$$16^2 + 63 = 65^2$$

$$20^2 + 21^2 = 29^2$$

# Diophantus

- Major work is the *Arithmetica*
- Fermat's theorem was inspired by reading Diophantus
- Diophantus worked mostly with fractions

“Find three squares such that the difference between the greatest and the middle has a given ratio to the difference between the middle and the least”



# Diophantus

## □ Diophantine equations

□ A typical Diophantine equation is to find two integers  $x$  and  $y$  such that their sum, and the sum of their squares, equal two given numbers  $A$  and  $B$ , respectively

□  $A = x + y$

□  $B = x^2 + y^2$

□ *In general*  $a^n + b^n = c^n$

# Fermat

- Born in 1601 A.D.
- Became interested in mathematical work by studying Greek mathematicians
- Main interest was in whole numbers these problems fascinated Fermat
- Never published any of his work
- Died in 1665 A.D.

# What Does this mean?

- Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

- It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.

Can you Solve these Equations?

$$3^2 + 4^2 = c^2$$

$$5^2 + 12^2 = c^2$$

$$2^3 + 2^3 = c^3$$

$$3^3 + 3^3 = c^3$$

$$2^4 + 2^4 = c^4$$

$$3^4 + 3^4 = c^4$$

Can you find solutions to the cubed equations in whole numbers?

Fermat claimed that there were none.

In fact, he claimed that for the general family of equations:

In whole numbers

$$x^n + y^n = z^n \text{ where } n > 2$$

it's impossible to find a solution

# 1670

- After 1670 – Fermat's son Samuel republished the work of Diophantus with some of his father's notes included. The only theorem Fermat left unproven was that  $x^n + y^n = z^n$  has no whole number solution for any power except  $n=2$ . Proofs were available for  $n=3$  and  $n=4$ , but not for  $n=5$  or greater.
- Leonhard Euler went through Fermat's work and found proofs however Fermat's Statement in the margin was hard to prove.
- Solutions were found for the case of  $n=3$  and  $n=4$
- Because this was the only one of Fermat's assertions to remain unproven for so long it took the name "Fermat's last proof"

# Sophie Germain 1808

- 1808 – Sophie Germain published work where she split the proof into two cases and showed that there are no solutions in which none of  $x$ ,  $y$ , or  $z$  is divisible by the exponent  $n$  if the prime exponent  $n$  satisfied the additional constraint that  $2n+1$  should also be prime. Fermat's theorem was now proved for up to 100 in the first case of Germain's proof.”
- 1825 – Lejeune Dirichlet gave a partial proof of the second case of  $n=5$  to the Paris Academy.
- After 1825 – Adrien-Marie Legendre finished Dirichlet's part of the proof of the second case  $n=5$  .
- Late 1820's – Gabriel Lamé proved the theorem for  $n=7$  .
- 1830 – Lamé used a complex number to factor  $x^n + y^n$  and proposed a draft of the full theorem to the Paris Academy. The proof did not work, but gained the interest of Ernst Kummer.
- After 1830 – Ernst Kummer gave a proof for any exponent that is a “regular” prime, but the proof did not work for other primes.

# Solved by

- Andrew Wiles
  - ▣ September 1994



# Sources

- "Pierre de Fermat -." *Wikipedia, the free encyclopedia*. Web. 9 Sept. 2009. <<http://en.wikipedia.org/wiki/Fermat>>.
- "Fermat's Last Theorem -." *Wikipedia, the free encyclopedia*. Web. 9 Sept. 2009. <[http://en.wikipedia.org/wiki/Fermat%27s\\_Last\\_Theorem](http://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem)>.
- "Pythagoras -." *Wikipedia, the free encyclopedia*. Web. 21 Sept. 2009. <<http://en.wikipedia.org/wiki/Pythagoras>>.
- Berlinghoff, William P. *Math through the ages a gentle history for teachers and others*. Washington, DC: Mathematical Association of America, 2004. Print.