Shapes by the Numbers: Coordinate Geometry

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Timeline for people developing analytic geometry

• 350BC - Menaechmus
• 262BC – 190BC – Apollonius
• 1323AD – 1382AD Nicole Oresme
• 1591AD – Francois Viete
• 1630AD – Pierre De Fermat
• 1637AD – Rene Descartes
• 1649AD – Frans van Schooten
• 1655 – John Wallis
Analytical Geometry

• Also known as coordinate geometry and Cartesian geometry
• Is the study of geometry using a coordinate system and the principles of algebra and analysis
• Simply put, it represents shapes by equations
• Contrasts Euclidean geometry which uses axioms and theorems to derive truths
• Is the foundation of most modern fields of geometry including algebraic, differential, discrete, and computational geometry. Also is widely used in Physics and engineering
• Is the bridge between geometry and algebra
• Without this bridge there would be no...
  – Calculus for science
  – CAT scans for medicine
  – Automated machine tools for industry
  – Computer graphics for art and entertainment
History of the Cartesian Coordinate System

• "Cartesian" refers to Rene Descartes who is credited with the invention of analytic geometry but Not the rectangular coordinate system. (his name was Cartesius in Latin).

• Egyptian surveyors used a rectangular grid to divide land into districts

• Similar methods were used by Roman and Greek mapmakers in early times
Early connections between equations and shapes

• Greece, 350 B.C. – Menaechmus (tutor of Alexander the Great) related curves to the solution of numerical proportions
  – The curves were formed by cutting a cone by a plane

• Groundwork for Apollonius’ exploration on conics (1 century later)
• Interested in locus questions: What points satisfy a given set of conditions, and do they form some kind of line or curve?
• Investigated complex locus questions and discovered that some result in conic sections
• His geometric figures were connected with numerical relationships by means of ratios and words BUT not exactly analytic geometry
Evolution of Algebraic Symbolism

• 14\textsuperscript{th} century - Nicole Oresme described a way of graphing the linear relationship between an independent variable and a dependent one.

• Late 16\textsuperscript{th} century - François Viète took a giant step toward focusing algebra on geometry problems - represented quantities with letters and relationships with equations.

• Beginning 17\textsuperscript{th} century - Fermat and Descartes gave the creative insight to connect algebra and geometry.
Pierre De Fermat (1601-1665)

- French Mathematician
- Interested in the locus problems of Apollonius
- Developed many of the key concepts of analytic geometry by about 1630
- Created a coordinate system
  - Plotted relationships between two unknown positive quantities, A and E
The Life of Rene Descartes (1596-1650)

• Born in France
• Mother died of tuberculosis when 1 year old
• His father was a member of parliament
• Rene earned his degree in Law and Science in accordance with his father’s wishes that he become a lawyer
• 2 years later he joined the army under the Dutch Republic
• He returned to school to study mathematics
De La Methode

- Discourse on the Method of Rightly Conducting Reason and Seeking Truth in the Sciences
- Covered optics, meteorology, and geometry
- Descartes appendix on geometry introduced the main ingredients for analytic geometry
La Geometrie

• The main ingredient for analytic geometry
• Same graphical devices as Fermat
  – The independent variable x marked off along a horizontal line and the dependent variable y represent by line segment making a fixed angle with the x segment
• Descartes emphasized that the angle choice was a matter of convenience not always a perfect right angle
La Geometrie Cont.

• View powers by defining a unit of length and the interpreting all quantities in term of that unit (i.e. $x^2, x^3$, and any power higher)
  – Previously Greeks only viewed powers as geometric dimensions
• This shift in representation allowed for the consideration of curves defined by functions containing various powers of an unknown
• He could graph these unknowns without a restriction of geometric dimension
The Death of Rene Descartes

- Died on February 11th, 1650 in Stockholm, Sweden
- Cause of death was pneumonia
- Worked as a teacher for the Queen Christina of Sweden
- Some believe his death was due to a lack of sleep compromising his immune system
  - Accustomed to working in bed until noon, he may have suffered a detrimental effect on his health due to Christina’s demands for early morning study
Descartes Impact

• Algebra lacked the level of rigor as geometry
• *Discourse on Method* was written in French
• Latin = Universal language of 17\textsuperscript{th} century
• Left out many proofs
  – He did not want to “deprive you of the pleasure of mastering it yourself”
  – Much work left for Frans Van Schooten
Frans Van Schooten (1615-1660)

- In 1649, translated *La Geometrie* into Latin
- Published an expanded version
- His work was published in four editions and eight times as long as the original
- Isaac Newton learned about analytic geometry through this work while developing the fundamental ideas of calculus
Cartesian Geometry

- Very well known by end of 17th century
- Still did not include the ordinate (vertical axis)
- Fermat usually considered the angle to be a right angle
- Only considered positive coordinates
- In 1650, John Wallis included negative coordinates
- The vertical axis we now use seems to have just developed over time with no one particular inventor
Mathematical Advancements

• Analytic Geometry was another part of a chain of mathematical ideas
• Symbolic Algebra ➔ Analytic Geometry
• Analytic Geometry ➔ Calculus
• Calculus ➔ Modern Physics
Sources

• Berlinghoff and Gouvea. Math Through the ages. Print.