Linear Thinking
Solving First Degree Equations

9/21/09

MAT 400

Chessa Horomanski
Jessica DiPaul

The Rhind Papyrus

- Named after A. Henry Rhind
- Collection of problems probably used for training young scribes in Ancient Egypt

Source: HistoryofScience.com

Sample Problem

0011 0010 1010 1101 0001 0100 1011

A quantity; its half and its third are added to it. It becomes ten.

- For us: $x + \frac{1}{2}x + \frac{1}{3}x = 10$
- A scribe is instructed to solve it as we would:

Divide 10 by $1 + \frac{1}{2} + \frac{1}{3}$

A Different Method

Example:

A quantity; its fourth is added to it. It becomes fifteen.

- Assume (posits) the quantity is 4
- Take 4 and add its fourth to it, you get 4+1=5, but we wanted 15
- We need to multiply what we got (5) by 3 to get what we wanted to get (15)
- Take our guess (4) and multiply by 3 to get an answer of 12

False Position

0011 0010 1010 1101 0001 0100 1011

- We posit an answer that we don't really expect to be the right one
  - Makes the computations easy
- Use the incorrect result find number we need to multiply our guess to get the correct answer

You try it!

0011 0010 1010 1101 0001 0100 1011

Use the method of false position to solve this problem.

A quantity and its ½ added together become 16. What is the quantity?

Assume 2. Take $2 + \frac{1}{2} \cdot 2 = 3$. We wanted 16 but we got 3. We need to multiply what we got (3) by $5\frac{1}{3}$ to get 16. Take our guess 2 and multiply it by $5\frac{1}{3}$

$5\frac{1}{3} \cdot 2 = 10\frac{2}{3}$ ⟵ **The quantity**

Symbols

0011 0010 1010 1101 0001 0100 1011

- Using symbols

$$Ax = B$$

$$A(kx) = k(Ax) = kB$$

- Only works on equations in the form of

$$Ax = B$$

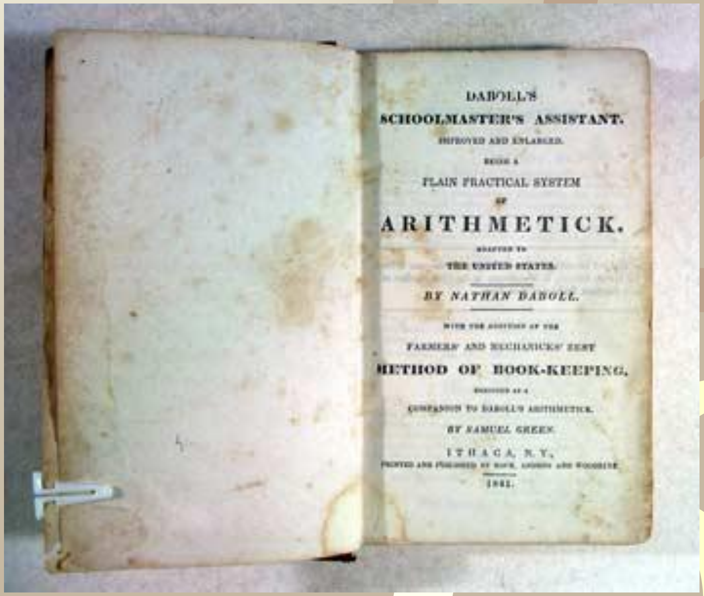
- Does not work on equations of the form

$$Ax + C = B$$

Double False Position

0011 0010 1010 1101 0001 0100 1011

- Effective method for solving linear equations that it continued to be used long after the invention of algebraic notations
- Requires no algebra
- Taught in arithmetic textbooks
  - *Daboll's Schoolmaster's Assistant*



Source: HarvestofHistory.org

Example of Double False Position

0011 0010 1010 1101 0001 0100 1011

A purse of 100 dollars is to be divided among four men A, B, C and D, so that B may have four dollars more than A, and C eight dollars more than B, and D twice as many as C. What is each one’s share of the money?

Modern Approach:

0011 0010 1010 1101 0001 0100 1011

Let x be the amount given to A. Then B gets x+4, C gets (x+4)+8=x+12 and D gets 2(x+12). Total is $100.

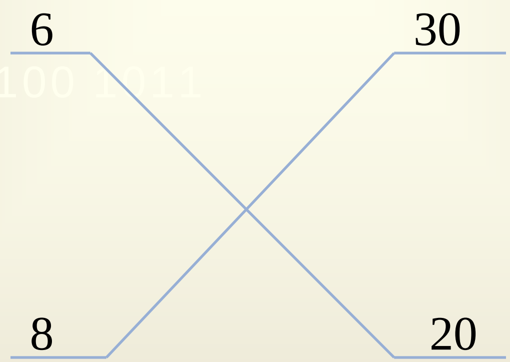
$$x + (x + 4) + (x + 12) + 2(x + 12) = 100$$

Daboll’s Method

0011 0010 1010 1101 0001 0100 1011

- Make a guess – assume A gets 6 dollars
  - Then B gets 10, C gets 18, and D gets 36
  - Adding them together we get $70, only $30 off
- Let’s try again
  - Assume A gets 8 dollars, B gets 12, C gets 20, and D gets 40
  - Total now is $80, still off by $20

MAGIC!!



- Cross multiply and subtract the differences to get 120
- Divide the difference of errors to get 10
- Divide 120 by 10 so the right choice for man A is $12
- Only works when both errors are same type

Isn’t this puzzling?? Why does it work?

0011 0010 1010 1101 0001 0100 1011

Let’s use some graphical thinking.

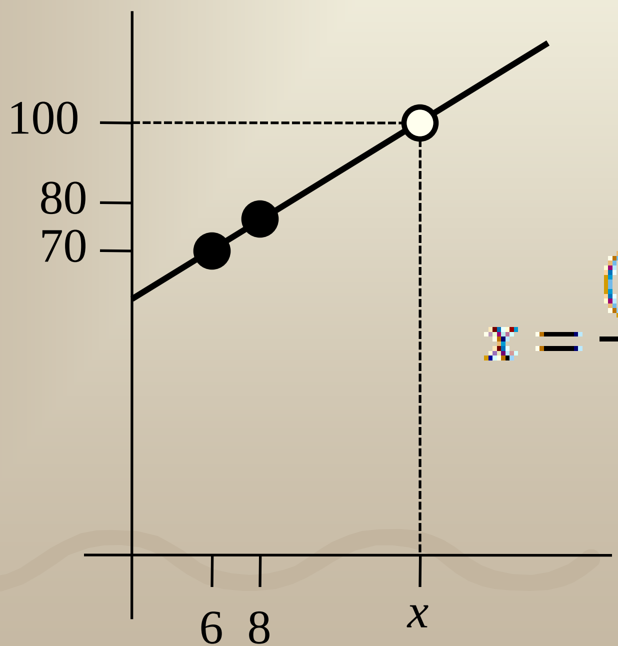
$$x + (x + 4) + (x + 12) + 2(x + 12) = 100$$

By simplifying the left side, the equation will be something of the form mx + b =100

We need 2 points.

We will use our guesses: (6, 70) and (8, 80)

We want to find *x* so that (*x*, 100) is on the same line.



$$\frac{100-70}{x-6}=\frac{100-80}{x-8}$$

$$x=\frac{(30\times 8)-(20\times 6)}{30-20}=12$$

0011 0010 1010 1101 0001 0100 1011

- Our way of understanding equations as lines is quite recent (17th century)
- Double false position is very old
- Change in the output is proportional to the change in the input → "linearity"

“Linear” and “Nonlinear”

<u>Linear</u>
Simple relation – a constant ratio – between changes in the input and output

<u>Nonlinear</u>
No such simple relation – very small changes in the input may produce huge changes in the output

- We use linear problems to find approx. solutions to nonlinear ones.
- The methods we use are based on the fundamental insight which served as the basis for the method of false position.

Timeline

1650 B.C.

0011 0010 1010 1101 0001 0100 1011

- Rhind Papyrus
    - -False position
    - -Double false position

Early 17th century

- Modern way of understanding equations as lines

Early 1800s

- *"Daboll's Schoolmaster's Assistant"* –most popular arithmetic book in America prior to 1850

References

0011 0010 1010 1101 0001 0100 1011

- Berlinghoff, William P. and Fernando Q. Gouvea. *Math Through the Ages*. Oxton House Publishers, Maine, 2002.
- "Daboll's Schoolmaster's Assistant." http://www.harvestofhistory.org/primary_source_detail.html?ps_id_69 (September 16, 2009).
- John Fauvel and Jeremy Gray, eds. *History of Mathematics: A Reader*. Macmillan Press Ltd., Basingstoke, 1988.
- Lucas N. H. Bunt, Phillip S. Jones, and Jack D. Bediant. *The Historical Roots of Elementary Mathematics*. Dover Publications, New York, 1976.
- "Mathematics/Logic Timeline: From Cave Paintings to the Internet." http://historyofscience.com/G21/timeline/index.php?category=Mathematics+%2F+Logic (September 16, 2009).