J. Bertrand, Solution d'un problème, Comptes Rendus de l'Académie des Sciences, Paris 105 (1887) p. 369.

THEORY OF PROBABILITY. — Solution of a problem; by Mr. J. Bertrand.

Suppose that two candidates A and B are in an election. The number of the voters is μ . A obtains m votes and is elected, B obtains $\mu - m$. Find the probability that, during the counting of the votes, the number of votes for A always exceeds those of his competitor.

The required probability is $(2m - \mu)/\mu$. The proof is based on the following formula which is easy to explain.

If $P_{m,\mu}$ indicates the number of combinations which, in the counting of the votes, are favorable to the required event, one has

$$P_{m+1,\mu+1} = P_{m,\mu} + P_{m+1,\mu}.$$

The general expression of $P_{m,\mu}$ results from this formula, but it seems probable that such a simple result could be shown in a more direct way.

If the number of the voters is 60, it is necessary that the elected candidate obtains 45 votes so that the probability of preserving the majority throughout the poll is equal to 1/2.