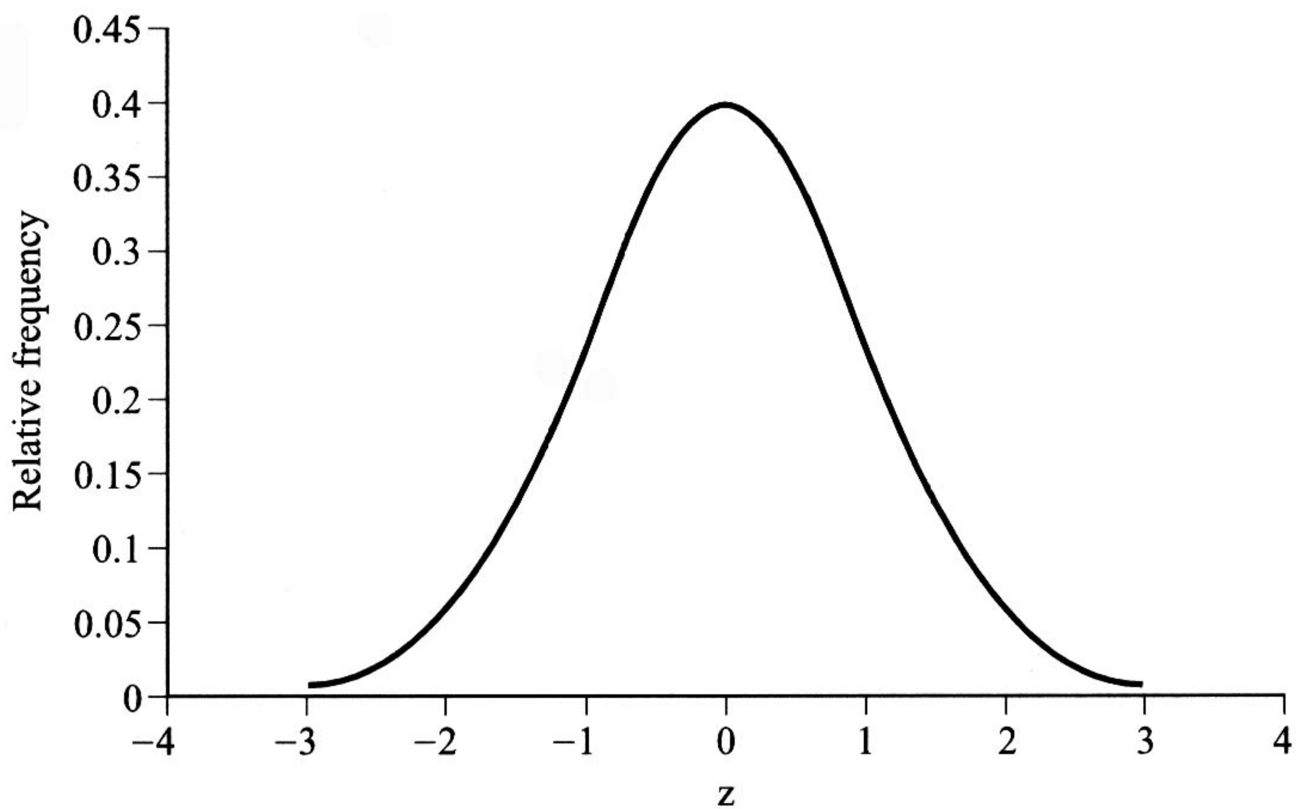
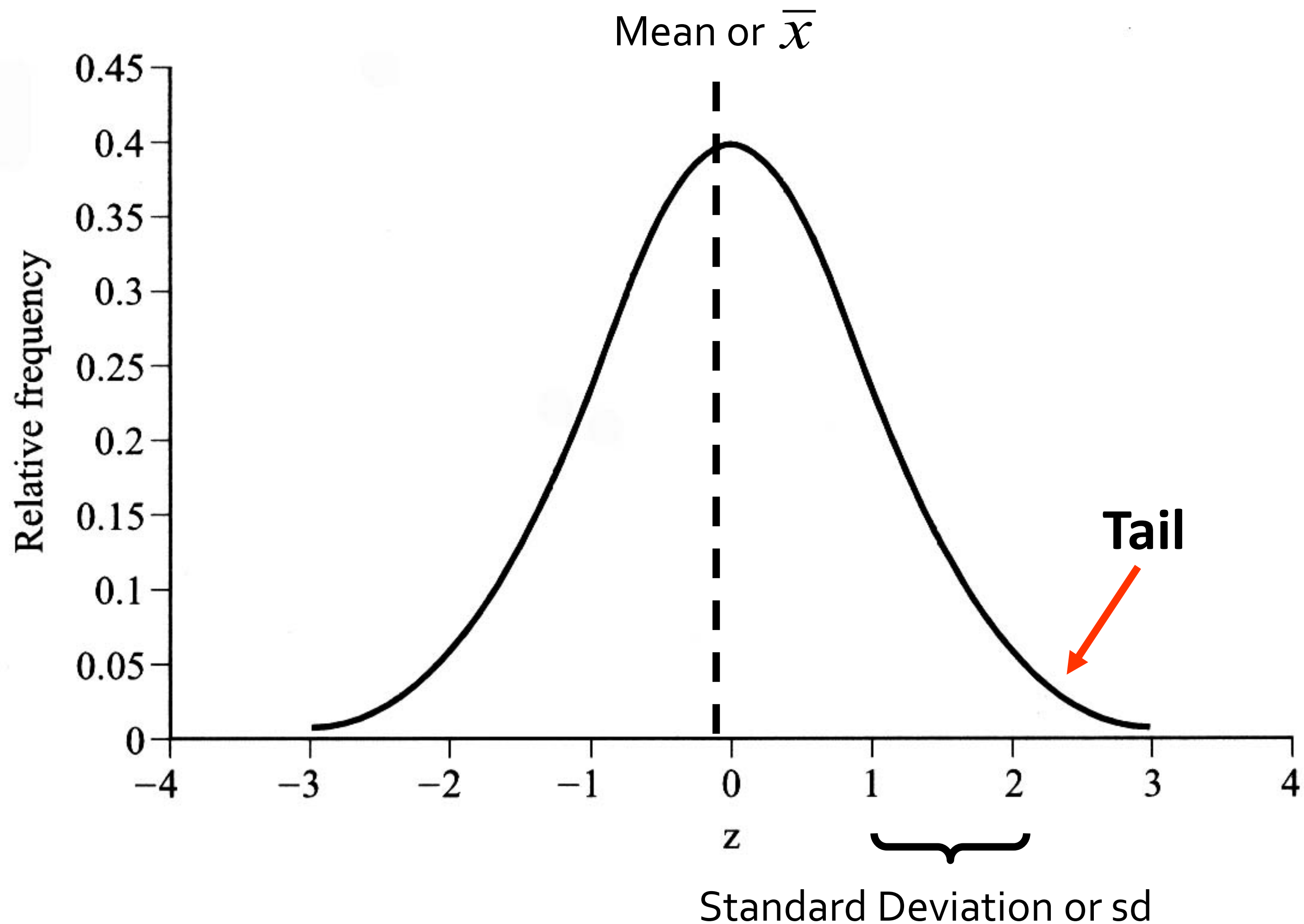
Continuous Probabilities:
Normal Distribution, Confidence Intervals for the Mean, and Sample Size

The Normal Distribution

Normal (Gaussian) distribution: a symmetric distribution, shaped like a bell, that is completely described by its mean and standard deviation.



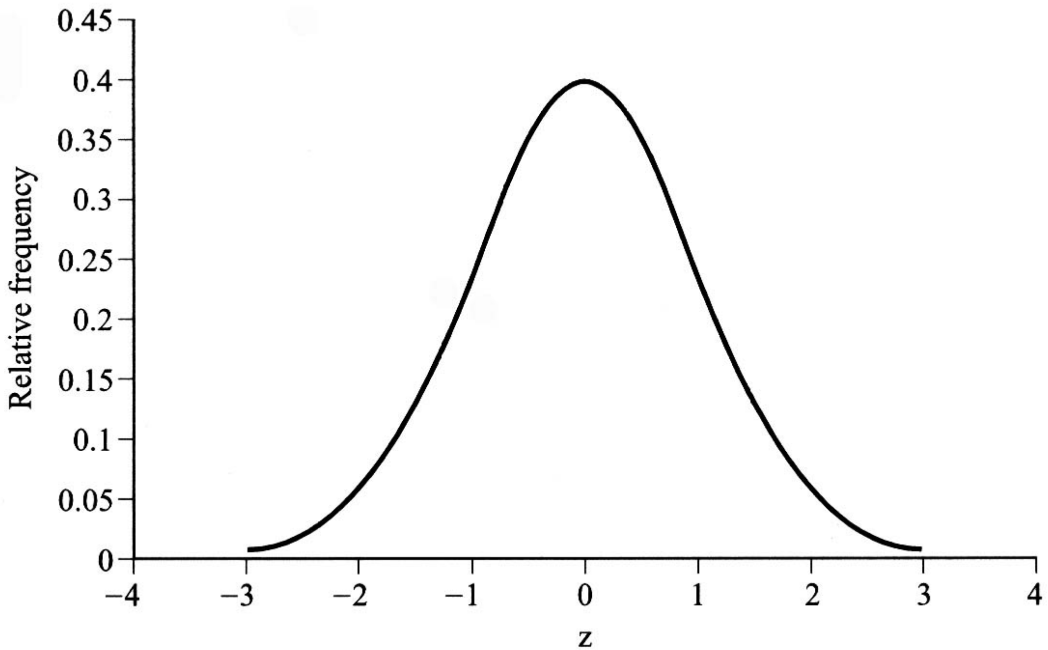


Every distribution has 2 tails.

- There are an infinite number of "normal" curves.

- To be useful, the normal curve is *standardized* to a mean of 0 and a standard deviation of 1.

- This is called a *standard normal curve*.



To use the standard normal curve, data must first be converted to *z-scores*.

Z-score: a transformation that expresses data in terms of standard deviations from the mean.

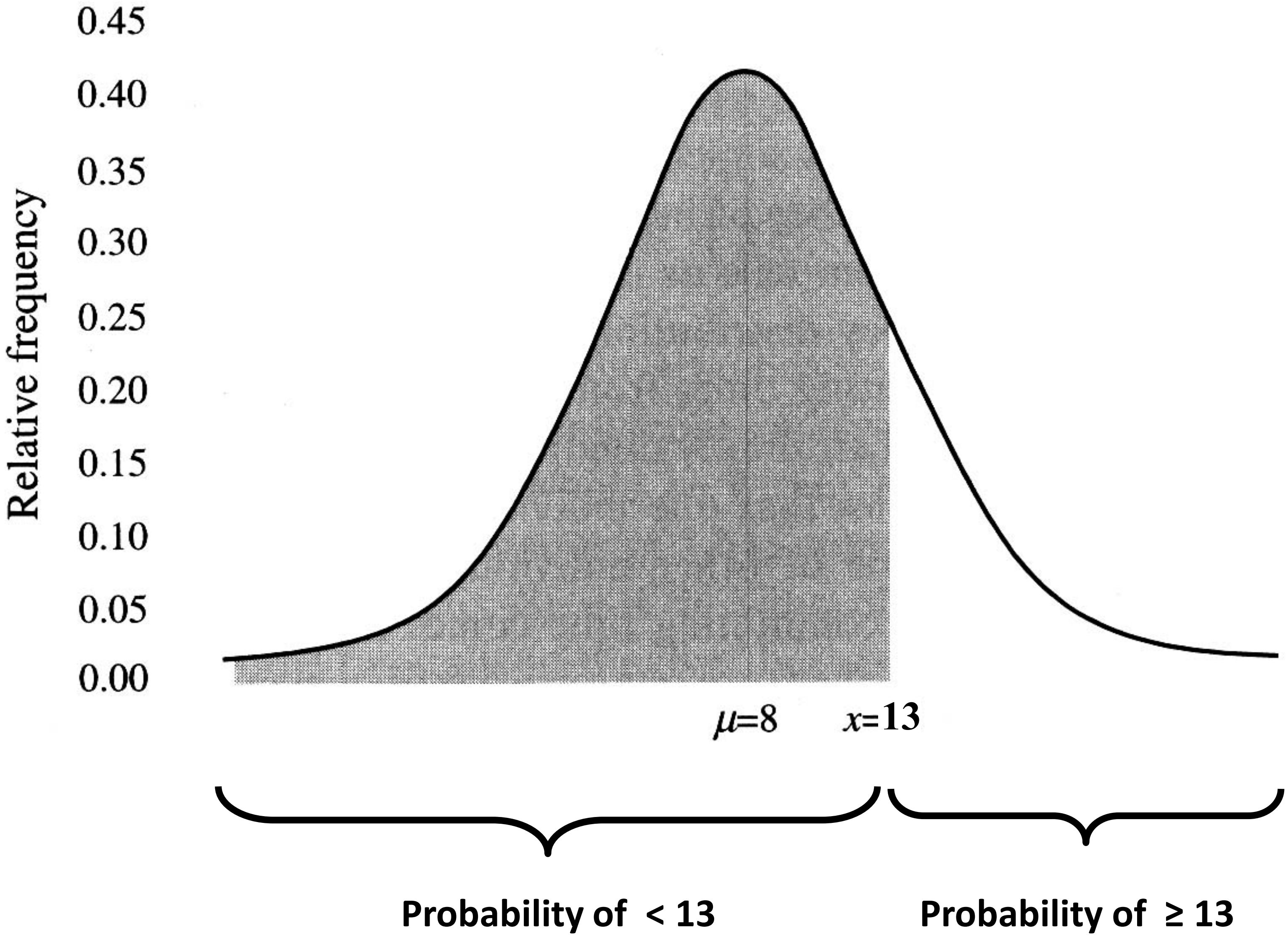
$$z = \frac{(x - \bar{x})}{s}$$

For example: We have a sample that has a mean of 8 and a standard deviation of 2.53. What is the z-score of an observation from this data set that has a value of 13?

$$z = \frac{(13 - 8)}{2.53} = 1.98$$

Therefore, a value of 13 in this data set is 1.98 standard deviations from the mean.

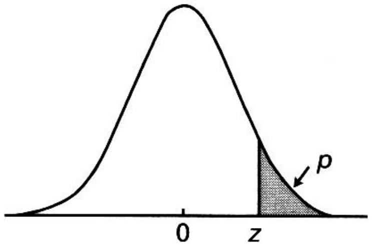
We can use the z-table to find out the probability of picking a number >= 13 from this data set.



Relative frequency
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
μ=8
x=13
Probability of < 13
Probability of ≥ 13

Standard Normal Probabilities

Table A.2

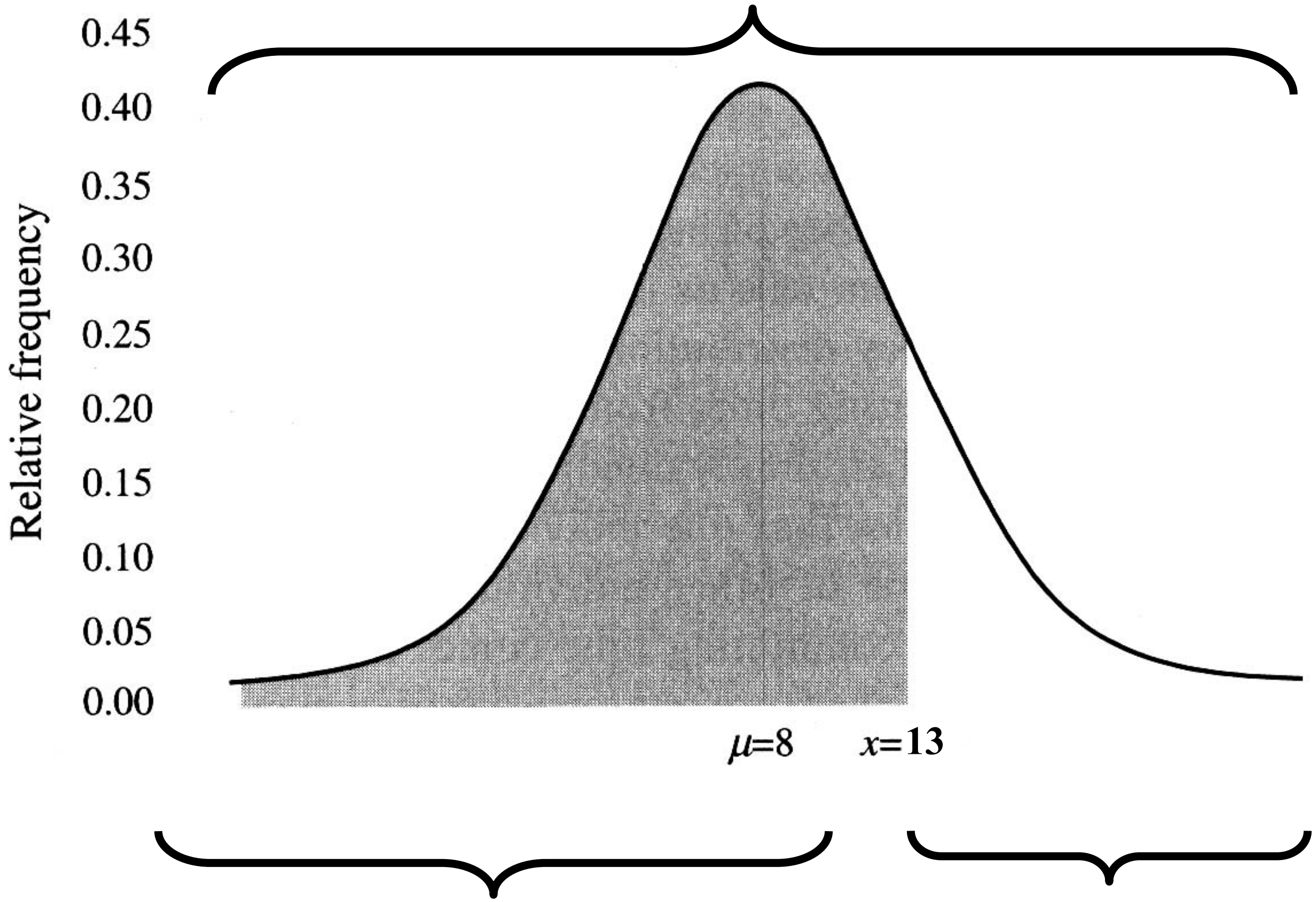


Note!

z	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

Adapted with rounding from Table II of Fisher and Yates 1974.

1.0 or 100%



p = (1 - 0.0239) = 0.9761

97.6% chance of picking a value < 13

p = 0.0239

2.4% chance of picking a value => 13

Probability density functions (e.g. normal distribution) are used to determine the probabilities that an event will or will not occur.

So for picking a value => 13:

97.6% chance that it will not occur.

- There is a 97.6% chance of picking a number less than 13 IF the mean is 8 and sd is 2.53.

2.4% chance that it will occur.

- There is a 2.4% chance of picking a number less than 13 IF the mean is 8 and sd is 2.53.

So if it is improbable that an event will occur (and a 2.4% chance IS improbable), and it DOES occur… that is of interest.

Confidence Intervals about the Mean

Any time a large number of independent, identically distributed observations are summed, the sum will have a normal distribution.

- *Independent* means that one observation does not influence the value of another observation.

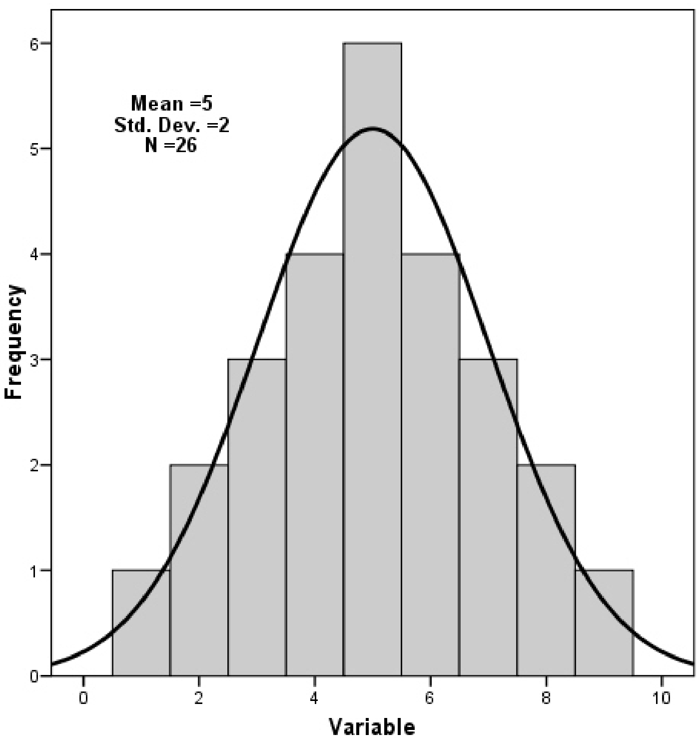
- *Identically distributed* means that each observation is from the same frequency distribution.

So if we take many samples and compute many means, the average of those means will be close to the true mean.

Experimental Data Set:

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 8, 8, 9

For these data the true mean is 5 and the true SD is 2.



$$n = 26$$

$$\bar{x} = \frac{130}{26} = 5$$

$$s = \sqrt{\frac{(1-5)^2 + (2-5)^2 \ldots (9-5)^2}{26-1}} = \sqrt{\frac{100}{25}} = 2$$

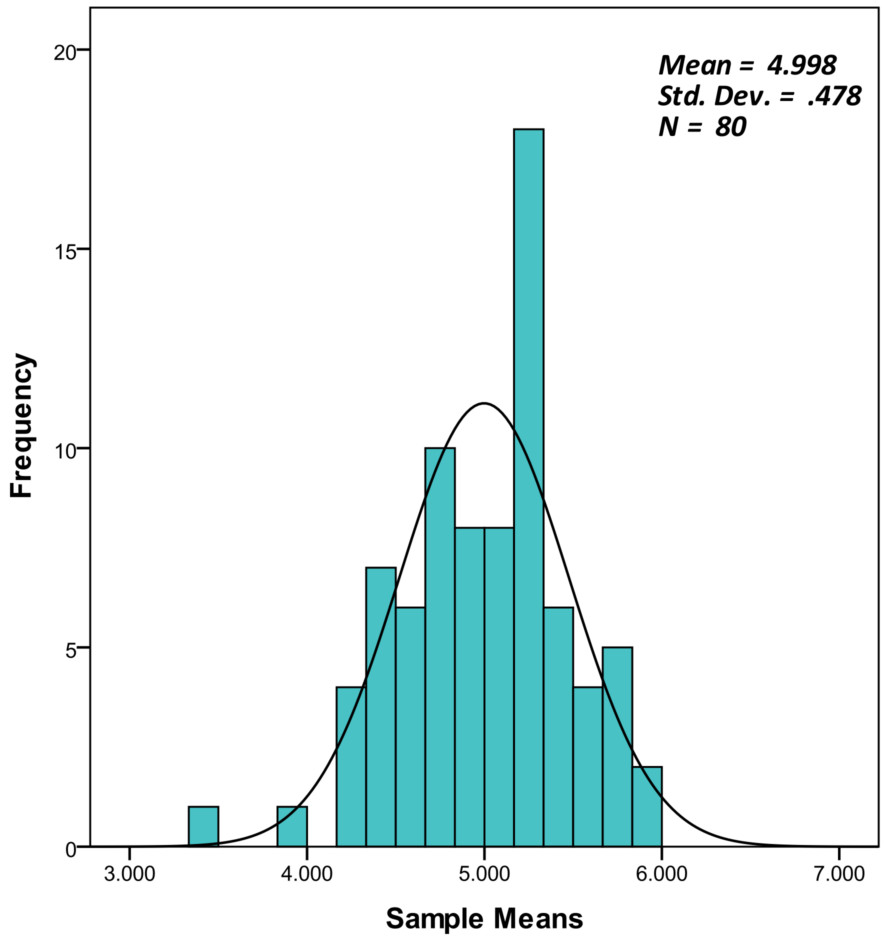
If we take a sample of these data, the mean of that sample should be close to the true mean (5).

Sample Data: 8, 7, 6, 6, 6, 5, 5, 4, 3, 3, 3, 1
$\Sigma x_i = 57$
$n = 13$

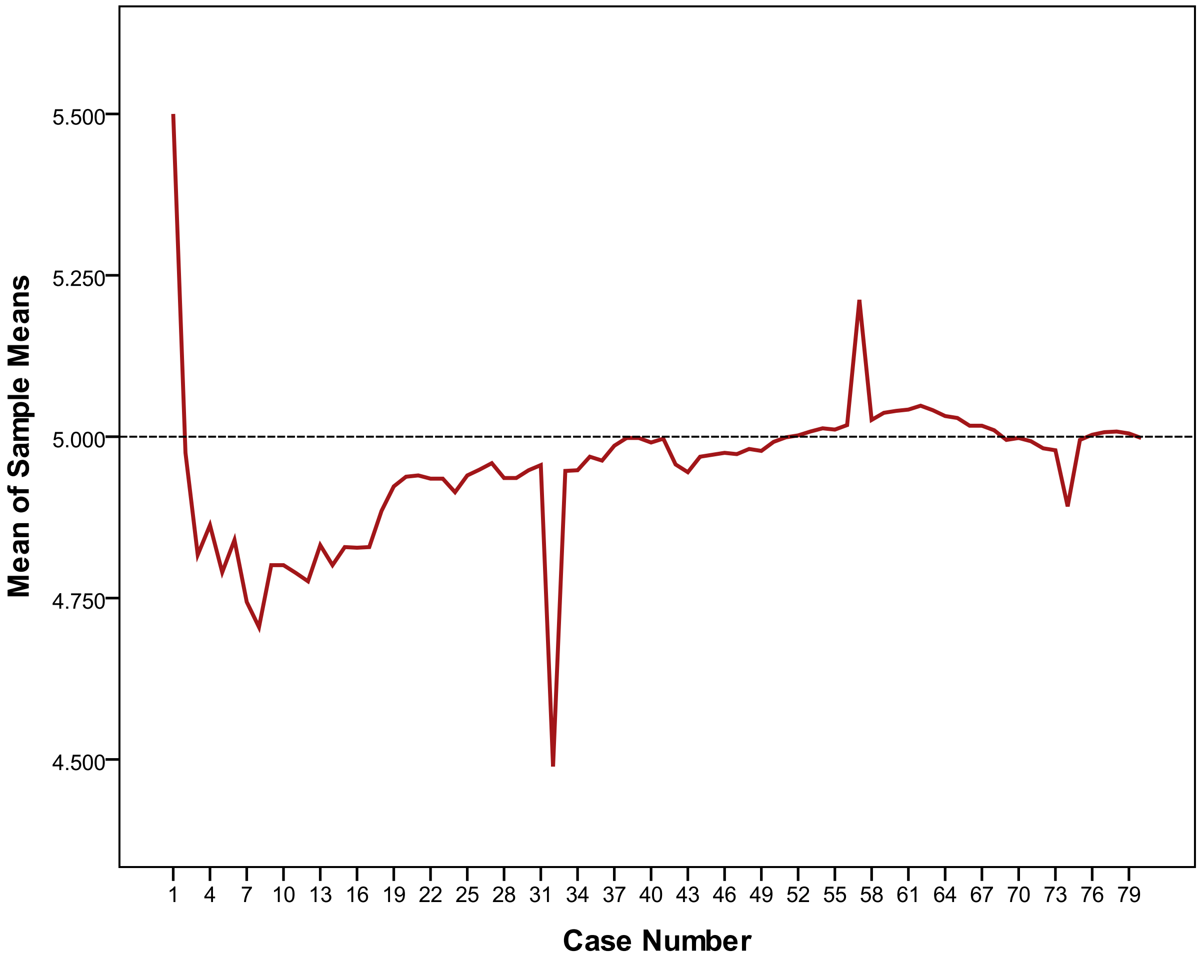
$$\bar{x} = \frac{57}{13} = 4.4$$

Time for the numbered chit experiment…

If a large number of samples are taken and we compute the means for each sample, those sample means should approach a normal distribution.



If as each new mean is calculated we calculate a running ‘mean of sample means’ and plot those as a line, as the number of sample means increases the line will approach the ‘true mean’… in this case, 5.00.



What this is telling us:

1. Samples follow a normal distribution.

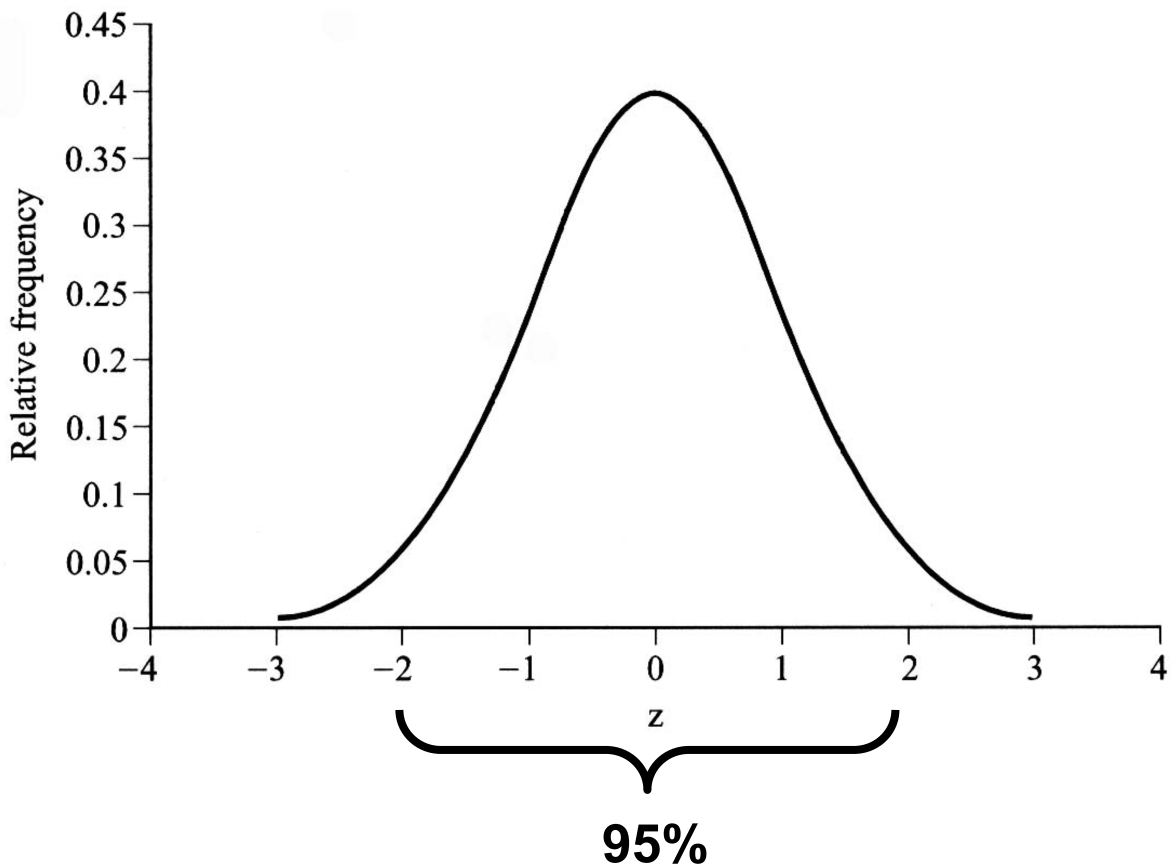
2. In normal distributions, most observations cluster close to the mean.

3. Therefore, our sample is likely to be close to the ‘true mean’.

4. However, we need to know HOW CLOSE.

Based on the normality of random sample means we can construct confidence intervals about a sample mean.

In a normal distribution, 95% of the data fall within 1.96 (approx. 2) standard deviations from the mean.



This implies that 95% of the time the *sample mean* lies with + or – 1.96 standard deviations from the *true mean.*

We can calculate this range using the equation:

$$prob\left[\left(\bar{x} - z_a \frac{s}{\sqrt{n}}\right) \leq \mu \leq \left(\bar{x} + z_a \frac{s}{\sqrt{n}}\right)\right] = 1 - \alpha$$

where z_α is taken from the z table. By convention we use either 95% (*z* = 1.96) or 99% (*z* = 2.58).

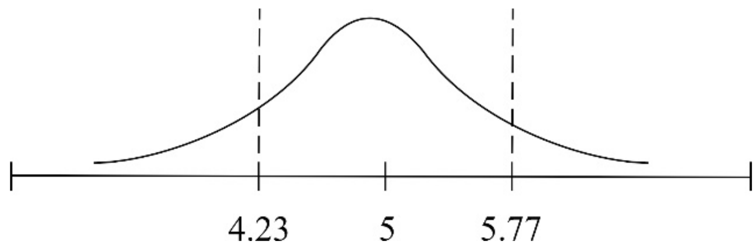
So for the example data set the 95% confidence intervals would be:

$$prob\left[\left(5 - 1.96\frac{2}{\sqrt{26}}\right) \leq \mu \leq \left(5 + 1.96\frac{2}{\sqrt{26}}\right)\right] = 0.95\ or\ 95\%$$

$$prob[4.23 \leq \mu \leq 5.77]$$

The equation gives you the actual location of the 95% confidence interval on the number line.

$$\boldsymbol{prob}\left[\left(5-1.96\frac{2}{\sqrt{25}}\right)\leq\boldsymbol{\mu}\leq\left(5+1.96\frac{2}{\sqrt{25}}\right)\right]=\boldsymbol{prob}[4.23\leq\boldsymbol{\mu}\leq 5.77]$$



If you want to use the ± notation you need to find the difference (or distance) between 5 and 5.77… which is 5 – 5.77 = 0.77. You can check the answer by 4.23 + 0.77 = 5. It would be written as:

$$\bar{x}(5) = \mathrm{CI}_{0.95} \pm 0.77$$

However, the normal distribution can only be used when the sample size is *large*…

For *smallish* sample sizes we use the *t distribution*.

T distribution: a symmetric distribution, more peaked than the normal distribution, that is completely described by its mean and standard deviation for *k degrees of freedom or df* (we will discuss this term in more detail later).

The *df* for confidence intervals is $n-1$. So for our example the *df* = 26-1 = 25.

Use a 2-tailed probability of 0.05 (1 – 0.95).

Again, we use the 2-tailed values since we are calculating confidence intervals that lie above and below the mean.

Critical Values of the t Distribution

Taken from Zar, 1984 Table B.3

Tails

ν	α(2): 0.50 α(1): 0.25	0.20 0.10	0.10 0.05	0.05 0.025	0.02 0.01	0.01 0.005	0.005 0.0025	0.002 0.001	0.001 0.0005
1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	[illegible]	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	$\boxed{2.060}$	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.682	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.682	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.682	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.681	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.681	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
41	0.681	1.303	1.683	2.020	2.421	2.701	2.967	3.301	3.544
42	0.680	1.302	1.682	2.018	2.418	2.698	2.963	3.296	3.538
43	0.680	1.302	1.681	2.017	2.416	2.695	2.959	3.291	3.532
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
45	0.680	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
47	0.680	1.300	1.678	2.012	2.408	2.685	2.946	3.273	3.510
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
49	0.680	1.299	1.677	2.010	2.405	2.680	2.940	3.265	3.500
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496

ν	α(2): 0.50 α(1): 0.25	0.20 0.10	0.10 0.05	0.05 0.025	0.02 0.01	0.01 0.005	0.005 0.0025	0.002 0.001	0.001 0.0005
52	0.679	1.298	1.675	2.007	2.400	2.674	2.932	3.255	3.488
54	0.679	1.297	1.674	2.005	2.397	2.670	2.927	3.248	3.480
56	0.679	1.297	1.673	2.003	2.395	2.667	2.923	3.242	3.473
58	0.679	1.296	1.672	2.002	2.392	2.663	2.918	3.237	3.466
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
62	0.678	1.295	1.670	1.999	2.388	2.657	2.911	3.227	3.454
64	0.678	1.295	1.669	1.998	2.386	2.655	2.908	3.223	3.449
66	0.678	1.295	1.668	1.997	2.384	2.652	2.904	3.218	3.444
68	0.678	1.294	1.668	1.995	2.382	2.650	2.902	3.214	3.439
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
72	0.678	1.293	1.666	1.993	2.379	2.646	2.896	3.207	3.431
74	0.678	1.293	1.666	1.993	2.378	2.644	2.894	3.204	3.427
76	0.678	1.293	1.665	1.992	2.376	2.642	2.891	3.201	3.423
78	0.678	1.292	1.665	1.991	2.375	2.640	2.889	3.198	3.420
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
82	0.677	1.292	1.664	1.989	2.373	2.637	2.885	3.193	3.413
84	0.677	1.292	1.663	1.989	2.372	2.636	2.883	3.190	3.410
86	0.677	1.291	1.663	1.988	2.370	2.634	2.881	3.188	3.407
88	0.677	1.291	1.662	1.987	2.369	2.633	2.880	3.185	3.405
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
92	0.677	1.291	1.662	1.986	2.368	2.630	2.876	3.181	3.399
94	0.677	1.291	1.661	1.986	2.367	2.629	2.875	3.179	3.397
96	0.677	1.290	1.661	1.985	2.366	2.628	2.873	3.177	3.395
98	0.677	1.290	1.661	1.984	2.365	2.627	2.872	3.175	3.393
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
105	0.677	1.290	1.659	1.983	2.362	2.623	2.868	3.170	3.386
110	0.677	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381
115	0.677	1.289	1.658	1.981	2.359	2.619	2.862	3.163	3.377
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
125	0.676	1.288	1.657	1.979	2.357	2.616	2.858	3.157	3.370
130	0.676	1.288	1.657	1.978	2.355	2.614	2.856	3.154	3.367
135	0.676	1.288	1.656	1.978	2.354	2.613	2.854	3.152	3.364
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
145	0.676	1.287	1.655	1.976	2.352	2.610	2.851	3.147	3.359
150	0.676	1.287	1.655	1.976	2.351	2.609	2.849	3.145	3.357
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
170	0.676	1.287	1.654	1.974	2.348	2.605	2.844	3.139	3.349
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
190	0.676	1.286	1.653	1.973	2.346	2.602	2.840	3.134	3.342
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
250	0.675	1.285	1.651	1.969	2.341	2.596	2.832	3.123	3.330
300	0.675	1.284	1.650	1.968	2.339	2.592	2.828	3.118	3.323
350	0.675	1.284	1.649	1.967	2.337	2.590	2.825	3.114	3.319
400	0.675	1.284	1.649	1.966	2.336	2.588	2.823	3.111	3.315
450	0.675	1.283	1.648	1.965	2.335	2.587	2.821	3.108	3.312
500	0.675	1.283	1.648	1.965	2.334	2.586	2.820	3.107	3.310
600	0.675	1.283	1.647	1.964	2.333	2.584	2.817	3.104	3.307
700	0.675	1.283	1.647	1.963	2.332	2.583	2.816	3.102	3.304
800	0.675	1.283	1.647	1.963	2.331	2.582	2.815	3.100	3.303
900	0.675	1.282	1.647	1.963	2.330	2.581	2.814	3.099	3.301
1000	0.675	1.282	1.646	1.962	2.330	2.581	2.813	3.098	3.300
∞	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758	2.8070	3.0902	3.2905

Therefore the calculation for the t distribution is:

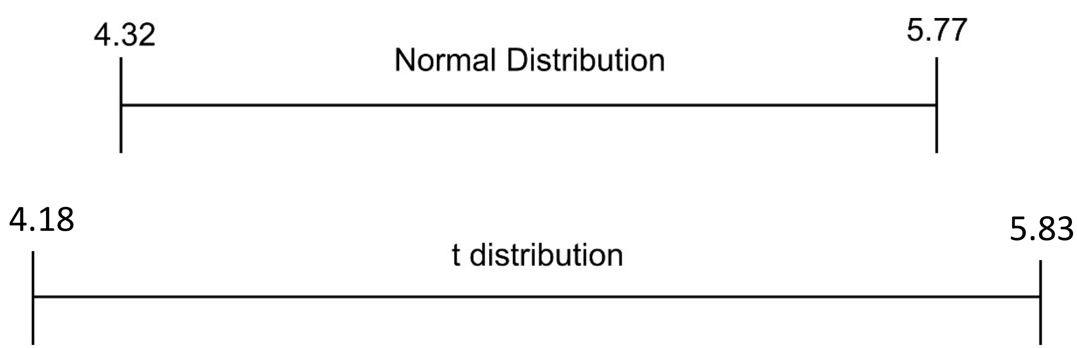
$$prob\left[\left(5 - 2.06\frac{2}{\sqrt{25}}\right) \leq \mu \leq \left(5 + 2.06\frac{2}{\sqrt{25}}\right)\right]$$

$$prob[4.18 \leq \mu \leq 5.83]$$

Note that the t distribution is more *conservative* (wider) than the normal distribution for small sample sizes.

$prob[4.23 \leq \mu \leq 5.77]$ **For normal distribution**

$prob[4.18 \leq \mu \leq 5.83]$ **For t distribution**



In SPSS

SPSS allows you to calculate any confidence interval but defaults to 95% intervals.

SPSS uses the equation:

$$\bar{x} + g_{(\alpha/2;0.05,d)} \times sd \le p \le \bar{x} + g_{(1-\alpha/2;0.5,d)} \times sd$$

$$where\ g_{(\alpha/2;0.05,d)}\ is\ \ from\ Odeh\ and\ Owen\ (1980, Table\ 1)$$

which is equivalent to the t distribution confidence intervals. So checking your work with SPSS is only good when calculating the t distribution confidence intervals.

Important:

The normal and t statistical test distributions have one thing in common: they are the sampling distribution of all possible means of samples of size n that could be taken from the population we are testing (Zar 1984, 99).

Sample Size and Test Power

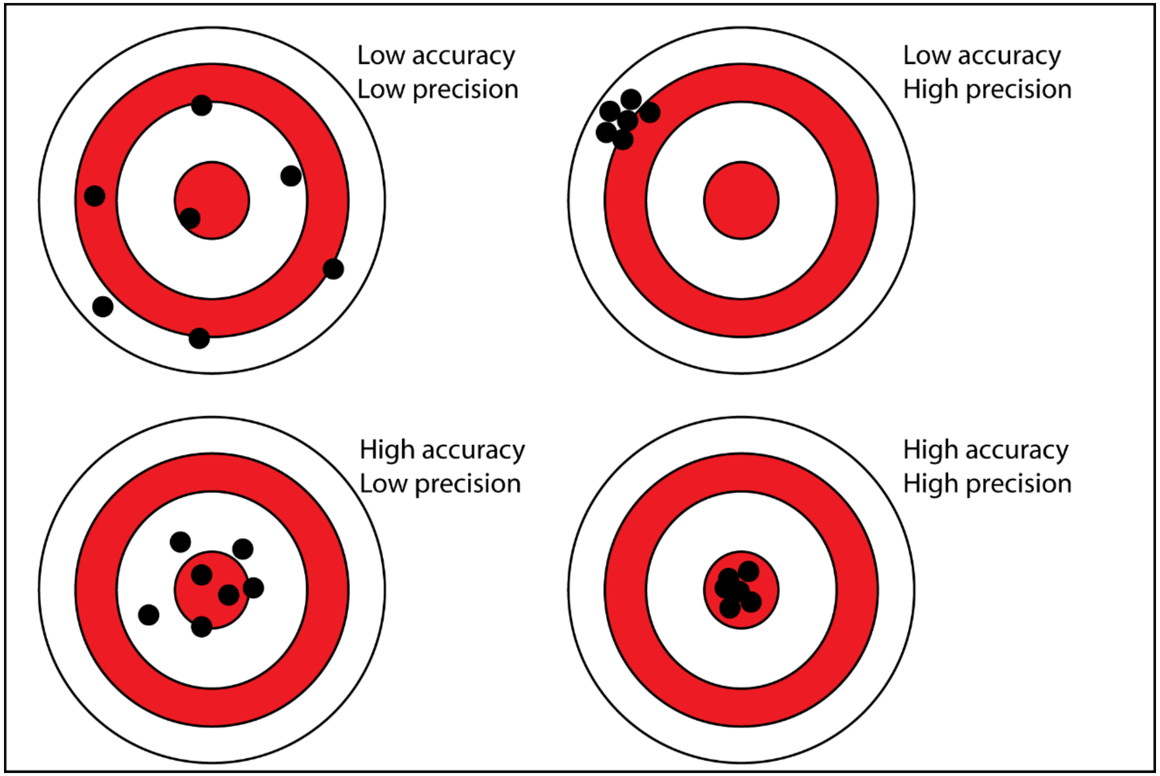
Sample Size and Estimating Population Parameters

The question often arises concerning how many samples are needed or what is the minimum sample size?

Many books state that 30 samples are the minimum to confidently perform a statistical analysis.

However, the minimum number of samples is related to the concept of precision and minimum detectible difference:

> If your measurements are in F°, smaller sample sizes may only allow one to detect differences with a precision of ±5°, while large sample sizes may allow for detection to less than 0.5°.



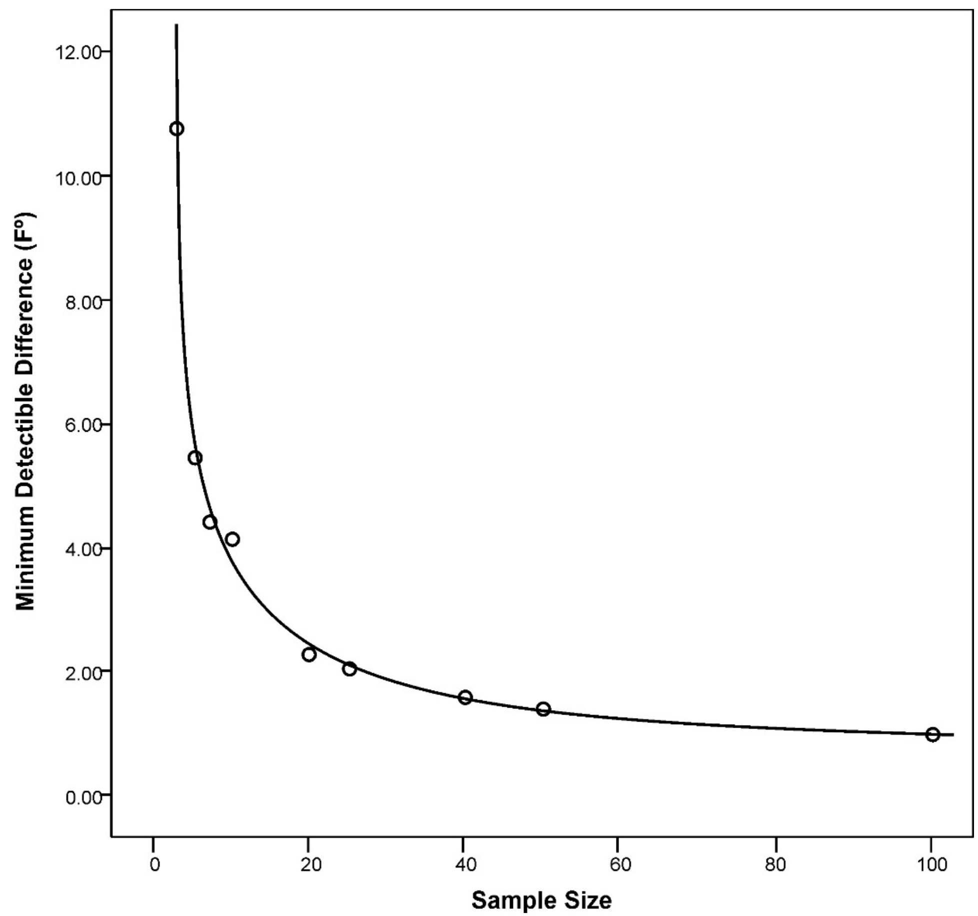
Accuracy: the closeness to which 'taken' measurement is to the 'true' measurement.

Precision: both the repeatability of a measurement AND the level of the measurement scale (e.g. measuring to 1 inch vs measuring to 0.001 inch).

The power to detect differences is not a linear function of sample size.

Also note that in this case after about 20 samples the power to detect smaller differences increases very slowly.

Gathering more than 20 samples in this case is a probably waste of time.



Source: Mimna, 2008

The minimum sample size required to detect a difference at a specific precision level can be estimated using the equation:

$$\delta \geq \sqrt{\frac{2s_p^2}{n}}(t_{\alpha,v} + t_{\beta(1),v})$$

where δ (delta) is the detectible difference, s^2_p is the sample variance, n is the sample size, and $t_{\alpha,v}$ and $t_{\beta(1),v}$ are the precision parameters taken from the t distribution table.

If this equation is solved several times for various sample sizes then a sample size function curve can be created.

For example, we are interested in determining if there is difference in the beak length (mm) between male and female humming birds. We want to be 90% sure of detecting a difference at a significance level (more on this later) of 0.05. The s^2 for the entire data set is 5.522.

Female Beak Length (mm)	Male Beak Length (mm)
36.6	37.5
33.5	36.0
37.7	40.3
36.7	38.4
36.9	38.7
33.1	36.1
32.3	35.0
38.2	39.3
36.3	38.0
35.3	37.9
34.5	36.8
36.6	38.8
34.6	37.3
35.8	38.4
39.0	42.9
34.4	37.1
35.2	37.0
36.8	39.1
35.9	37.9
34.1	37.4
40.7	43.8

From this table we choose values bounding our 90% confidence level (1 - 0.90 or 0.10) at our sample size (42). Since we are calculating upper and lower intervals, use the 2-tailed probabilities.

Critical Values of the *t* Distribution

Taken from Zar, 1984 Table B.3

Tails

ν	α(2): 0.50	0.20	0.10	0.05	0.02	0.01	0.005	0.002	0.001
	α(1): 0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.682	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.682	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.682	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.682	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.682	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.681	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.681	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.681	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.681	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
41	0.681	1.303	1.683	2.020	2.421	2.701	2.967	3.301	3.544
42	0.680	$\boxed{1.302}$	1.682	$\boxed{2.018}$	2.418	2.698	2.963	3.296	3.538
43	0.680	1.302	1.681	2.017	2.416	2.695	2.959	3.291	3.532
44	0.680	1.301	1.680	2.015	2.414	2.692	2.956	3.286	3.526
45	0.680	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
46	0.680	1.300	1.679	2.013	2.410	2.687	2.949	3.277	3.515
47	0.680	1.300	1.678	2.012	2.408	2.685	2.946	3.273	3.510
48	0.680	1.299	1.677	2.011	2.407	2.682	2.943	3.269	3.505
49	0.680	1.299	1.677	2.010	2.405	2.680	2.940	3.265	3.500
50	0.679	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
52	0.679	1.298	1.675	2.007	2.400	2.674	2.932	3.255	3.488
54	0.679	1.297	1.674	2.005	2.397	2.670	2.927	3.248	3.480
56	0.679	1.297	1.673	2.003	2.395	2.667	2.923	3.242	3.473
58	0.679	1.296	1.672	2.002	2.392	2.663	2.918	3.237	3.466
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
62	0.678	1.295	1.670	1.999	2.388	2.657	2.911	3.227	3.454
64	0.678	1.295	1.669	1.998	2.386	2.655	2.908	3.223	3.449
66	0.678	1.295	1.668	1.997	2.384	2.652	2.904	3.218	3.444
68	0.678	1.294	1.668	1.995	2.382	2.650	2.902	3.214	3.439
70	0.678	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
72	0.678	1.293	1.666	1.993	2.379	2.646	2.896	3.207	3.431
74	0.678	1.293	1.666	1.993	2.378	2.644	2.894	3.204	3.427
76	0.678	1.293	1.665	1.992	2.376	2.642	2.891	3.201	3.423
78	0.678	1.292	1.665	1.991	2.375	2.640	2.889	3.198	3.420
80	0.678	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
82	0.677	1.292	1.664	1.989	2.373	2.637	2.885	3.193	3.413
84	0.677	1.292	1.663	1.989	2.372	2.636	2.883	3.190	3.410
86	0.677	1.291	1.663	1.988	2.370	2.634	2.881	3.188	3.407
88	0.677	1.291	1.662	1.987	2.369	2.633	2.880	3.185	3.405
90	0.677	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
92	0.677	1.291	1.662	1.986	2.368	2.630	2.876	3.181	3.399
94	0.677	1.291	1.661	1.986	2.367	2.629	2.875	3.179	3.397
96	0.677	1.290	1.661	1.985	2.366	2.628	2.873	3.177	3.395
98	0.677	1.290	1.661	1.984	2.365	2.627	2.872	3.175	3.393
100	0.677	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
105	0.677	1.290	1.659	1.983	2.362	2.623	2.868	3.170	3.386
110	0.677	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381
115	0.677	1.289	1.658	1.981	2.359	2.619	2.862	3.163	3.377
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
125	0.676	1.288	1.657	1.979	2.357	2.616	2.858	3.157	3.370
130	0.676	1.288	1.657	1.978	2.355	2.614	2.856	3.154	3.367
135	0.676	1.288	1.656	1.978	2.354	2.613	2.854	3.152	3.364
140	0.676	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
145	0.676	1.287	1.655	1.976	2.352	2.610	2.851	3.147	3.359
150	0.676	1.287	1.655	1.976	2.351	2.609	2.849	3.145	3.357
160	0.676	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
170	0.676	1.287	1.654	1.974	2.348	2.605	2.844	3.139	3.349
180	0.676	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
190	0.676	1.286	1.653	1.973	2.346	2.602	2.840	3.134	3.342
200	0.676	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
250	0.675	1.285	1.651	1.969	2.341	2.596	2.832	3.123	3.330
300	0.675	1.284	1.650	1.968	2.339	2.592	2.828	3.118	3.323
350	0.675	1.284	1.649	1.967	2.337	2.590	2.825	3.114	3.319
400	0.675	1.284	1.649	1.966	2.336	2.588	2.823	3.111	3.315
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1000	0.675	1.282	1.646	1.962	2.330	2.581	2.813	3.098	3.300
∞	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758	2.8070	3.0902	3.2905

From the t-table.

$$\delta = \geq \sqrt{\frac{2(5.522)}{5}}(2.018 + 1.302) = 4.93mm$$

$$\delta = \geq \sqrt{\frac{2(5.522)}{10}}(2.018 + 1.302) = 3.49mm$$

$$\delta = \geq \sqrt{\frac{2(5.522)}{15}}(2.018 + 1.302) = 2.85mm$$

$$\delta = \geq \sqrt{\frac{2(5.522)}{20}}(2.018 + 1.302) = 2.48mm$$

$$\delta = \geq \sqrt{\frac{2(5.522)}{50}}(2.018 + 1.302) = 1.56mm$$

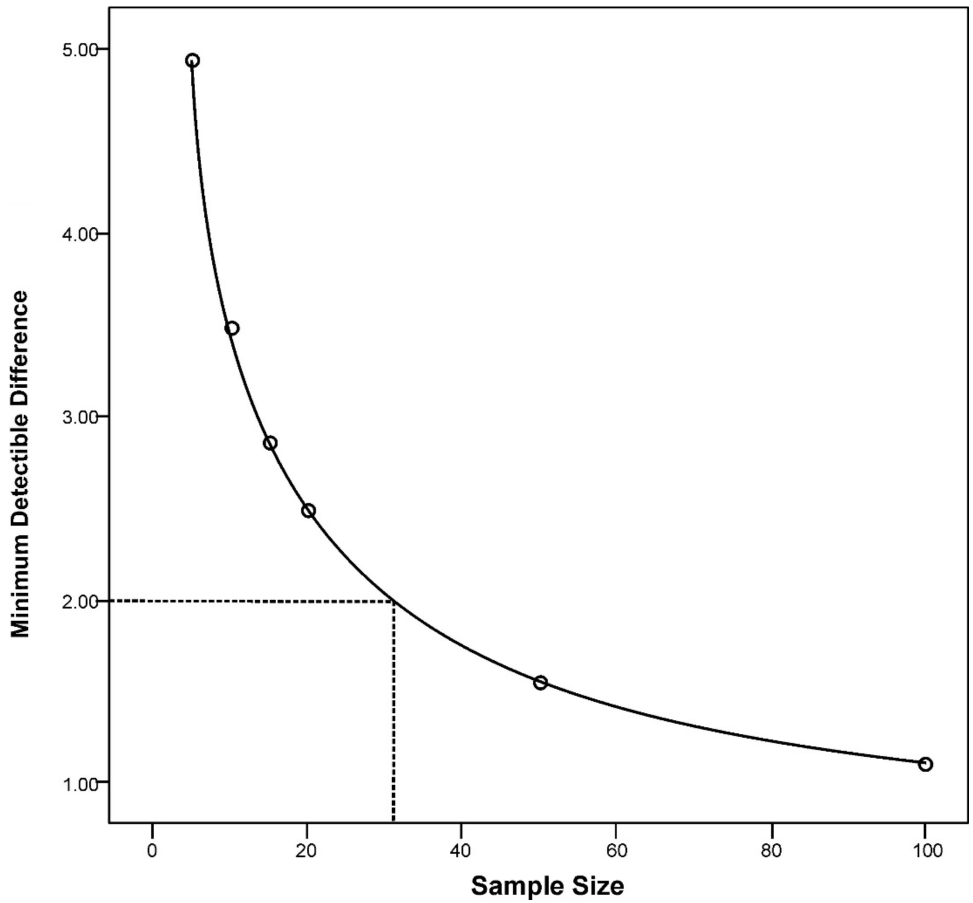
$$\delta = \geq \sqrt{\frac{2(5.522)}{100}}(2.018 + 1.302) = 1.1mm$$

Notice that all we are changing is the sample size, and that the rate of change in the minimum detectible difference decreases.

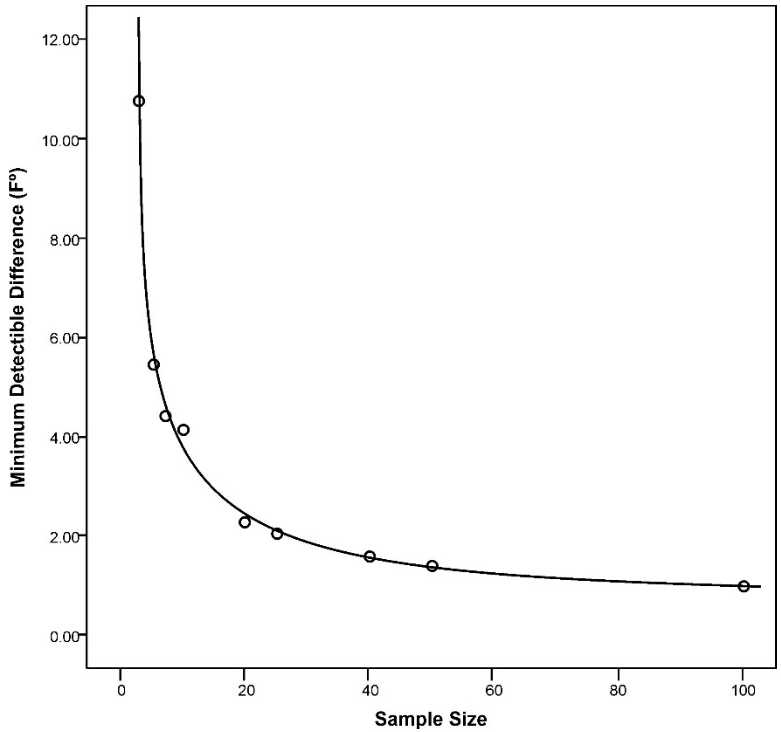
So with a sample of 5 we can only detect differences of $\geq$ 4.93 mm.

With a sample of 100 we can now detect differences down to $\geq$ 1.1 mm.

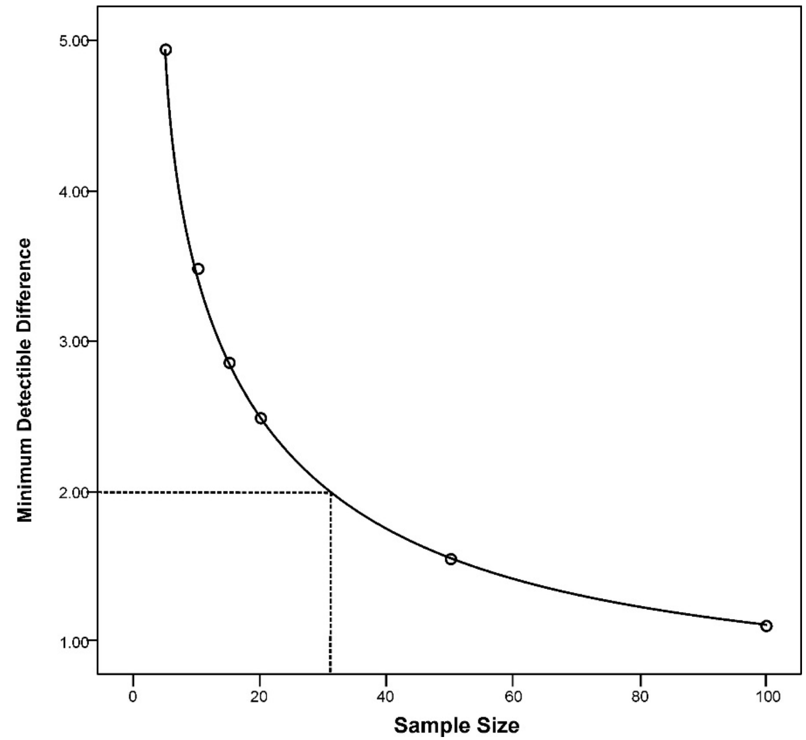
So if we need to detect a difference of at least 2mm, we'd have to have a sample size of about 32.



Mimna (2008)

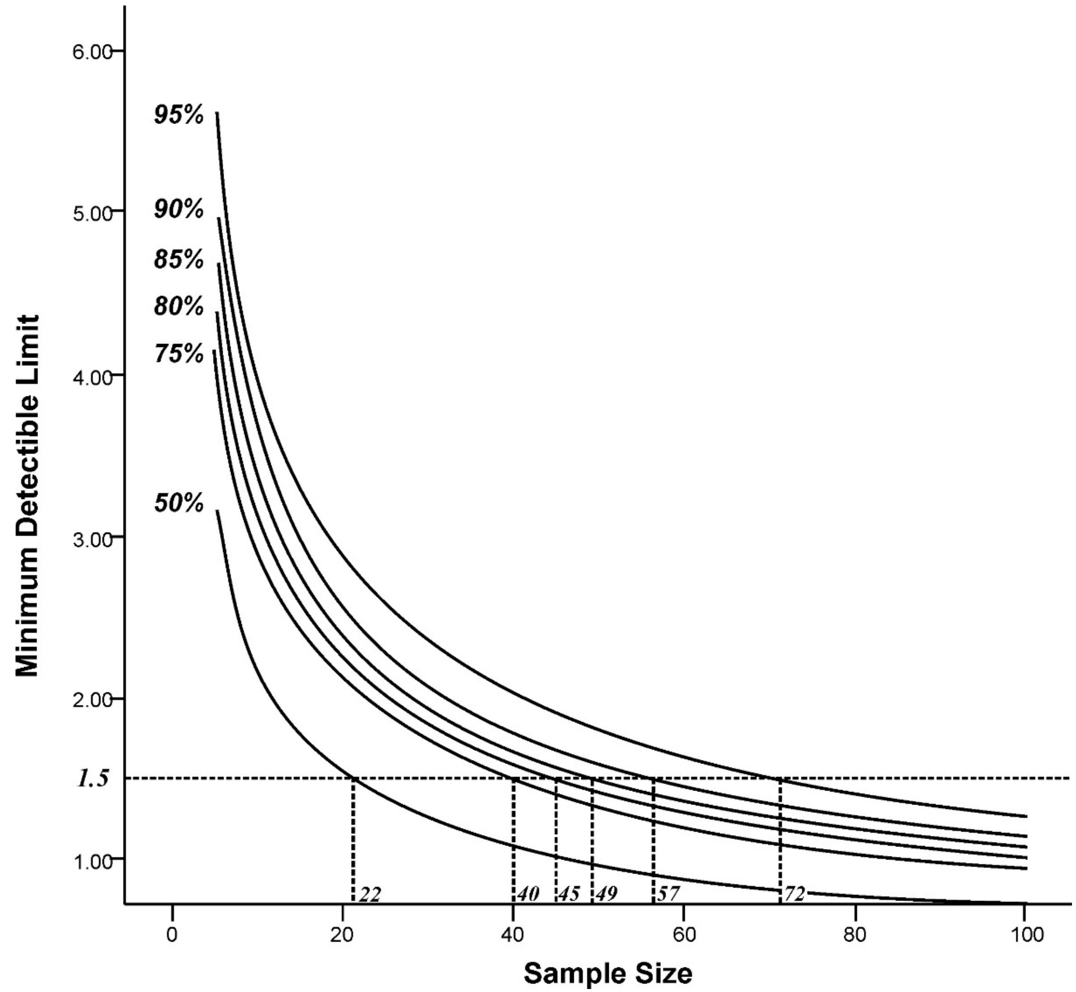


Marr (example)



Note that the curves for the two examples are different… there is no hard and fast minimum sample size number.

As the confidence limits are relaxed, the minimum sample size decreases. So in this case, if we wanted a minimum detectible value of 1.5, at 50% confidence we would need a sample of 22, at 75% confidence we would need a sample of 40, and at 95% confidence we would need a sample of 72.



Minimum sample size is a function of:

- The precision (in measured units, e.g. years, F°, etc…).
- The confidence level required.
- The variance of the data set.

Ultimately though, the minimum sample size *should* be based on our research precision requirements.