Statistical Equations
Quantitative Methods (GEO 441)

Statistic	*Equation*
Mean	$\mu = \dfrac{\sum_{i=1}^{n} x_i}{n}$
Median Location	$\dfrac{n}{2}$
Sample Variance	$s^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$
Sample Standard Deviation	$s = \sqrt{\dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$
Z-Score	$z = \dfrac{(x - \bar{x})}{s}$
Skewness	$k_3 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^3}{ns^3}$
Kurtosis	$k_4 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{ns^4} - 3$
Mean Confidence Interval (Normal Distribution, substitute t probabilities for t distribution)	$pr\left[\left(\bar{x} - z_\alpha \dfrac{s}{\sqrt{n}}\right) \le p \le \left(\bar{x} + z_\alpha \dfrac{s}{\sqrt{n}}\right)\right] = 1 - \alpha$
One-Sample z Test (gives probability directly based on the z table)	$z = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

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Statistic		*Equation*
One-Sample t Test $df = (n-1)$ **(gives probability based on the t table)**		$t = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
Two-Sample t Test $df = (n_1 + n_2 - 2)$		$t = \dfrac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$
Pooled Variance	where	$s_p = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
Mann-Whitney U Test $df = (n_1, n_2)$		$U = n_1 n_2 + \dfrac{n_1(n_1+1)}{2} - R_1$ $U' = n_1 n_2 - U$
F Test for Equality of Variances $df = (n_1 - 1, n_2 - 1)$		$F = \dfrac{s_1^2}{s_2^2}$ *or* $F = \dfrac{s_2^2}{s_1^2}$
Kolmogorov-Smirnov Normality Test		$D_i = \left\| rel\ F_i - rel\ \hat{F}_i \right\|$ *and* $D_i' = \left\| rel\ F_{i-1} - rel\ \hat{F}_i \right\|$
W/S (Range) Normality Test		$q = \dfrac{w}{s}$
D'Agostino Normality Test		$D = \dfrac{T}{\sqrt{n^3 SS}}$
	where	$T = \sum \left(i - \dfrac{n+1}{2} \right) X_i$

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Parametric AOV

$$F = \frac{BSS/k-1}{WSS/N-k}$$

$$df = (k-1, n-k)$$

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^2 - C$$

$$BSS = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} X_{ij}\right)^2}{n_i} - C$$

where $C = \frac{(\sum\sum X_{ij})^2}{N}$

and $WSS = TSS - BSS$

Non-Parametric AOV (Kruskal-Wallis)

$$df = (n_1, n_2 \ldots n_x)$$

$$H = \left(\frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i}\right) - 3(N+1)$$

Correction for tied ranks $C = 1 - \frac{\sum_i (t_i^3 - t_i)}{N^3 - N}$

Corrected H Statistic $H_c = \frac{H}{C}$

Pearson's Correlation Coefficient

$$df = (n-2)$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$$

$$\sum x^2 = \sum X^2 - \frac{\left(\sum X\right)^2}{n}$$

$$\sum y^2 = \sum Y^2 - \frac{\left(\sum Y\right)^2}{n}$$

$$\sum xy = \sum XY - \frac{\left(\sum X\right)\left(\sum Y\right)}{n}$$

Significance Test for r

$$t = \frac{r}{s_r} \quad where \quad s_r = \sqrt{\frac{1-(r)^2}{n-2}}$$

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Statistic	*Equation*
Spearman's Rank Correlation	$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n^3 - n}$
Significance test for r_s	$r_s \sqrt{n-1}$
Cramér's Dichotomous Correlation $df = (r-1)(c-1)$	$\phi = \frac{f_{11}f_{22} - f_{12}f_{21}}{\sqrt{C_1C_2R_1R_2}}$
Mean Center	$\bar{X}_{Coord} = \frac{\sum_{i=1}^{n} X_i}{n}$ $\bar{Y}_{Coord} = \frac{\sum_{i=1}^{n} Y_i}{n}$
Standard Distance **(***using the mean center coordinates***)**	$S_D = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n} + \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n}}$
Quadrat Analysis	$s^2 = \frac{\sum_{i=1}^{i=m}(x_i - \bar{x})^2}{m-1}$
where	$VMR = \frac{s^2}{\bar{x}}$
Normal Approximation	$z = \left(\sqrt{\frac{m-1}{2}}\right)(VMR - 1)$
X^2 Contingency Analysis $df = (r-1)(c-1)$	$\chi^2 = \sum\sum \frac{(f_{ij} - \hat{f}_{ij})^2}{\hat{f}_{ij}}$
where	$\hat{f}_{ij} = \frac{(Row_i)(Column_j)}{n}$

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Simple Linear Regression		$\hat{y}_i = a + bx_i$
Slope		$b = \frac{\sum xy}{\sum x^2}$
	where	$\sum xy = \sum X_i Y_i - \frac{\left(\sum X_i\right)\left(\sum Y_i\right)}{n}$
		$\sum x^2 = \sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n}$
y-intercept (constant)		$a = \overline{Y} - b\overline{X}$
Total Sum of Squares		$TSS = \sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n}$
Regression Sum of Squares		$RSS = \frac{\left(\sum xy\right)^2}{\sum x^2}$
Error Sum of Squares		$ESS = TSS - RSS$
Coefficient of Determination		$r^2 = \frac{RSS}{TSS}$
F-Test $df = v - 1, n - 2$		$F = \frac{RSS/1}{ESS/(n-2)}$
t-Test $df = n - 1$		$t = \frac{b}{s_b}$
	where	$s_b = \sqrt{\frac{s_e^2}{(n-1)s_x^2}}$

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Statistic	***Equation***
Converting Time to an Azimuth	$a = \frac{(360^0)(X)}{k}$
Mean Azimuth (Angle) *Sin +, Cos + : the mean angle is computed directly.* *Sin +, Cos - : the mean angle = 180 – ϑr.* *Sin -, Cos - : the mean angle = 180 + ϑr.* *Sin -, Cos + : the mean angle = 360 - ϑr.*	$X = \frac{\sum_{i=1}^{n} \cos_a}{n}$ $Y = \frac{\sum_{i=1}^{n} \sin_a}{n}$ $\cos \bar{a} = \frac{X}{r}$ $\sin \bar{a} = \frac{Y}{r}$ $r = \sqrt{X^2 + Y^2}$ $\theta_r = \arctan\left(\frac{\sin \bar{a}}{\cos \bar{a}}\right)$
Rayleigh's z Test $df = n$	$z = nr^2$
X^2 Test for Uniformity on a Circle $df = k - 1$	$\chi^2 = \sum \frac{(f_i - \hat{f}_i)^2}{\hat{f}_i}$ $\hat{f}_i = \frac{Sample\ Size}{Categories}$
Watson's U2 Test $df = n_1, n_2$	$U^2 = \frac{n_1 n_2}{N^2}\left[\sum d_k^2 - \frac{(\sum d_k)^2}{N}\right]$ $N = n_1 + n_2$
Runs Test on a Circle **(***Normal Approximation***)**	$Z = \frac{u'+1-\frac{2n_1n_2+N}{N}}{\sqrt{\frac{2n_1n_2(2n_1n_2-N}{N^2(N-1)}}}$