Transportation Systems as Networks

Using graph analyses we are interested in measuring such things as:

1. Traffic generated by nodes.
2. Flow along links.
3. Degree of accessibility and connectivity.
4. Spatial extent of a network.
5. Network association along a route.
6. Influence of one place on other places on a route or in a network.

Graph theory – reduces transport networks to a mathematical matrix whereby:

Edge: Line segment (link) between locations.

- Example: roads, rail lines, etc…

Vertex: Location on the transportation network that is of interest (node).

- Example: towns, road intersections, etc…

In transportation analysis graphs are ALWAYS finite… there are always constraining boundaries.

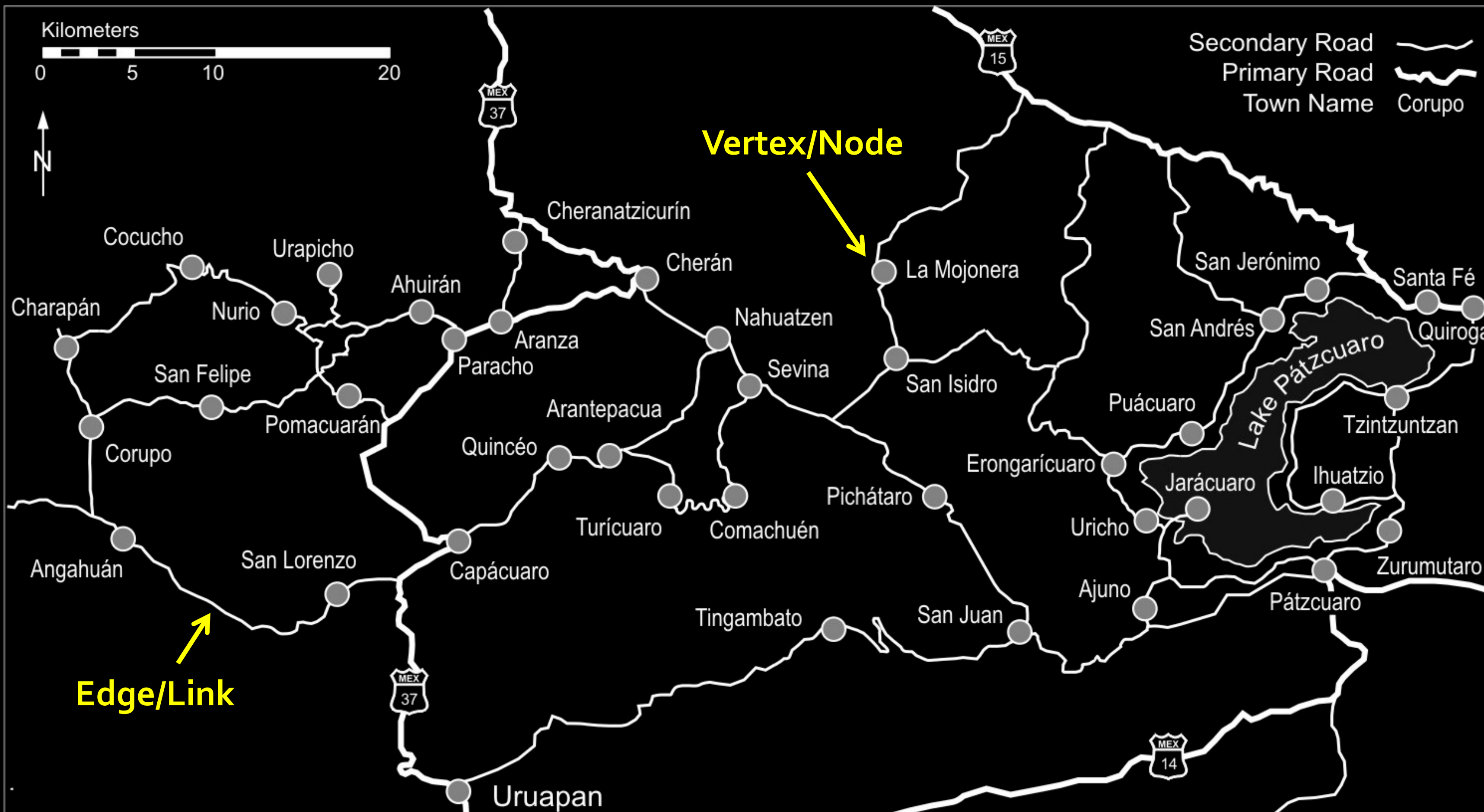
***Node* – a location on a transportation route that has the capacity to generate traffic (flow).**

***Link* – the connection between 2 nodes along which flow occurs.**

***Route* – a series of connected links.**

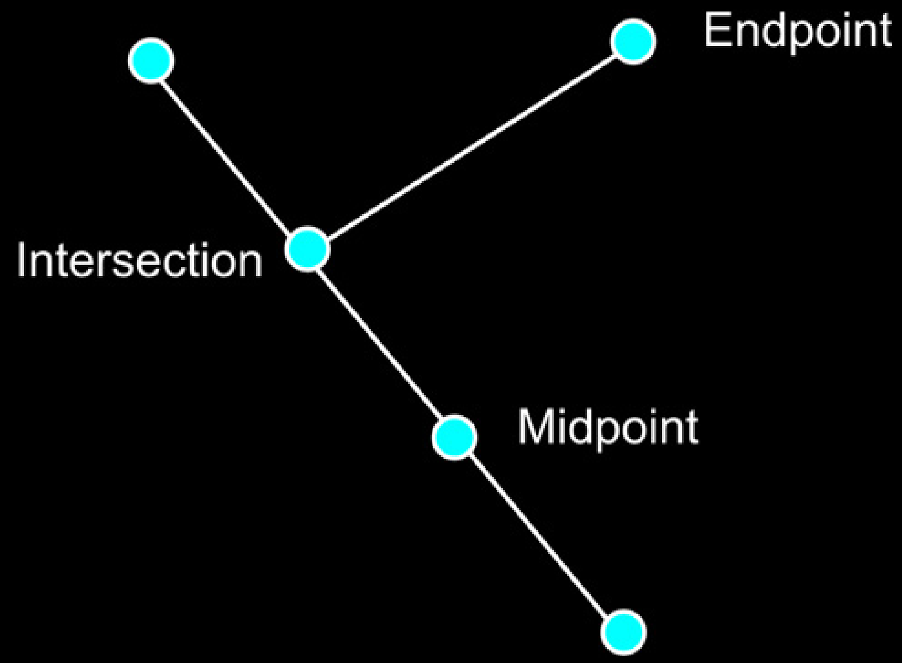
***Network* – a system of nodes and links. May consist of several modal types (road, rail, etc…), but is typically of a single mode type.**

Road System: Meseta Tarasca, Michoacán, Mexico



A few words about vertices (nodes):

1. Vertices must occur where 3+ edges meet.
2. The endpoints of edges are vertices.
3. Vertices can occur mid-link.



Directed graph – direction of flow is explicit.

Undirected graph – no flow direction implied.

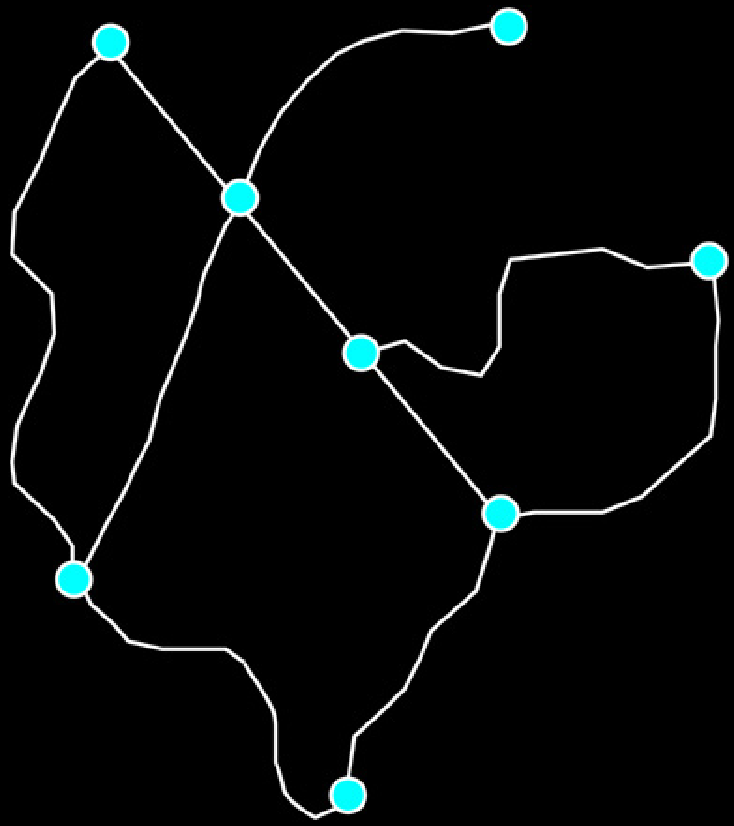
Loop – flow from a node into itself.

Planar – graphs where all links (edges) meet at nodes (vertices).

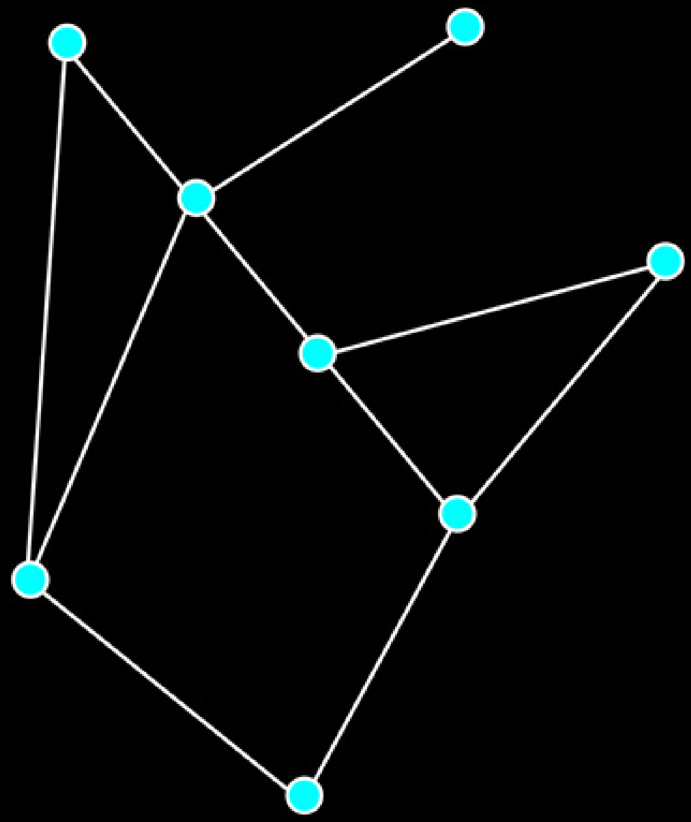
Non-planar – graphs where links (edges) may cross each other.

Element – a graph cell (dyad).

Real World Road System



Planar Network

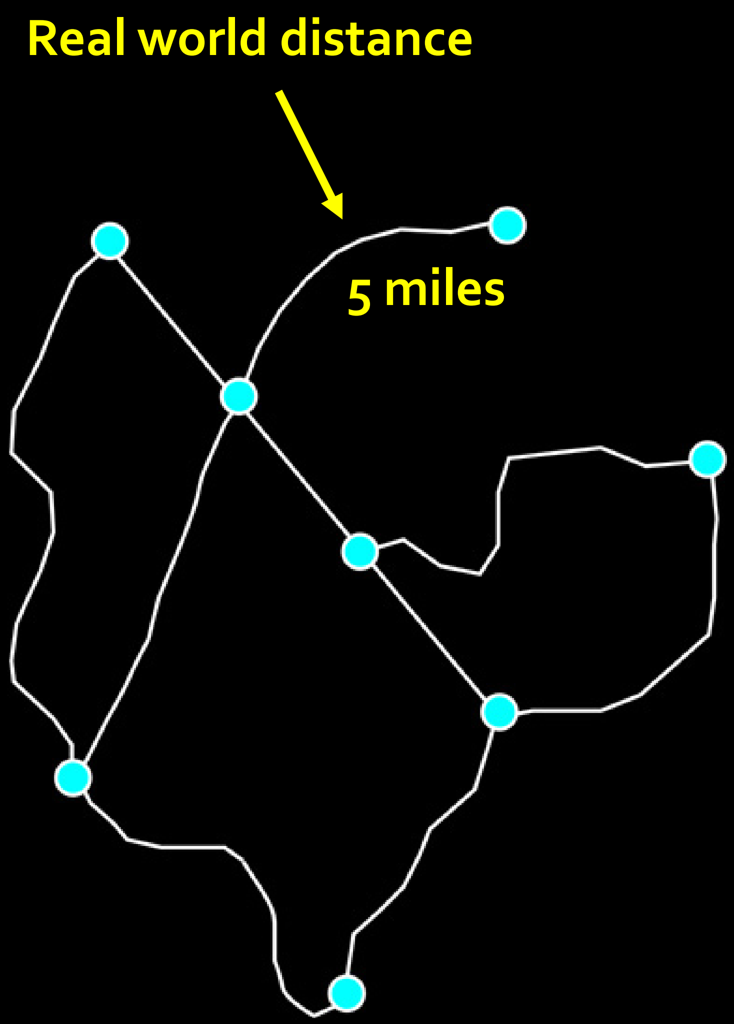


Topologic (geometric) distance – **a unit-less measure where the distance between nodes is coded as a single step.**

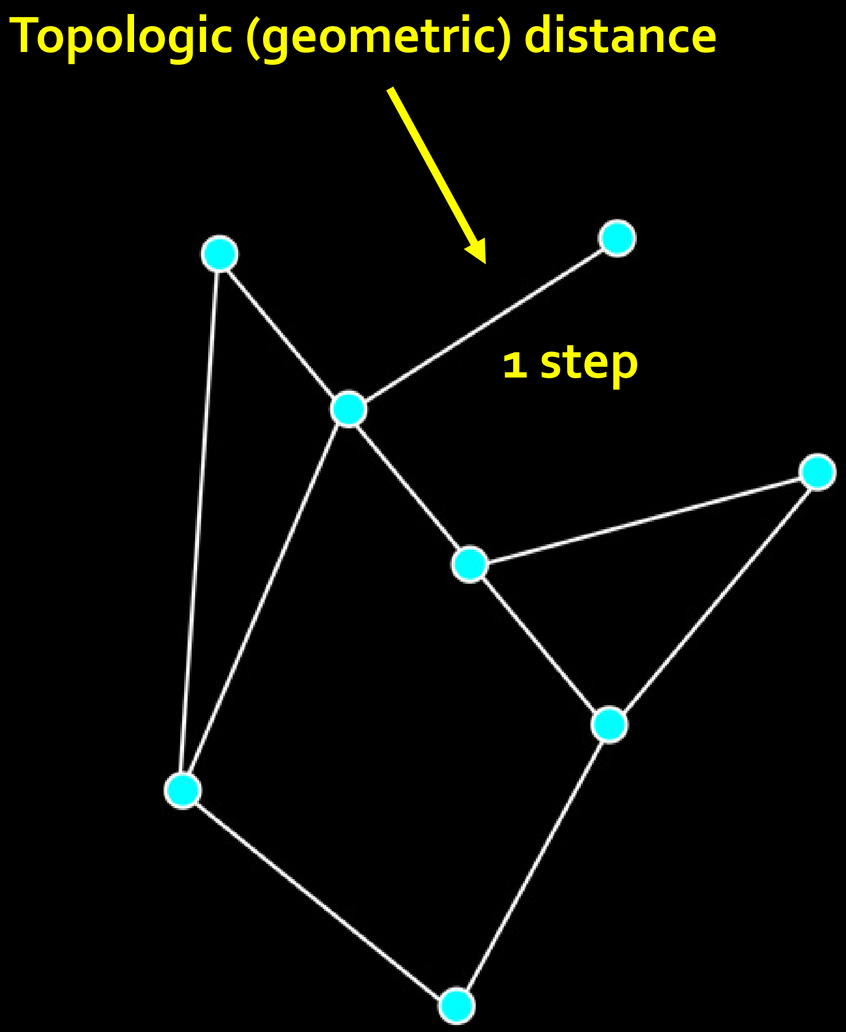
- **Distance is ONLY implied (e.g. more steps = longer distance.**

- **Real world distances are not used.**

- **Essentially removes the influence of distance.**



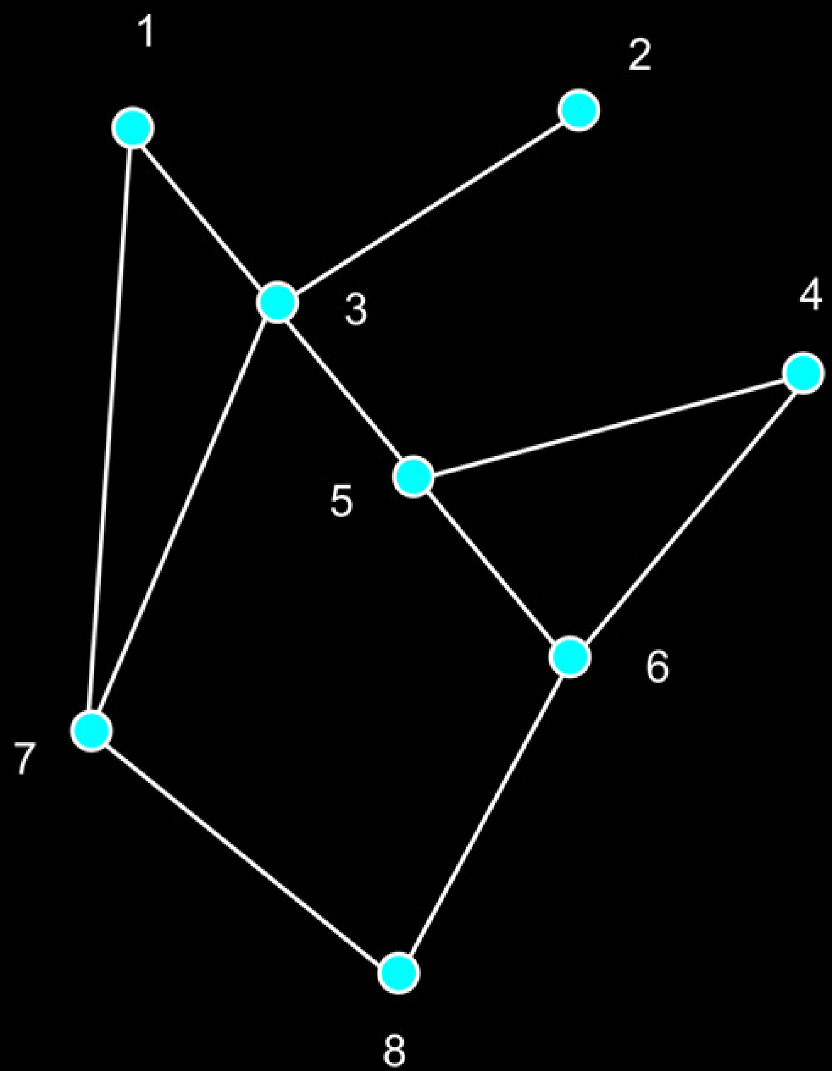
Real world distance
5 miles



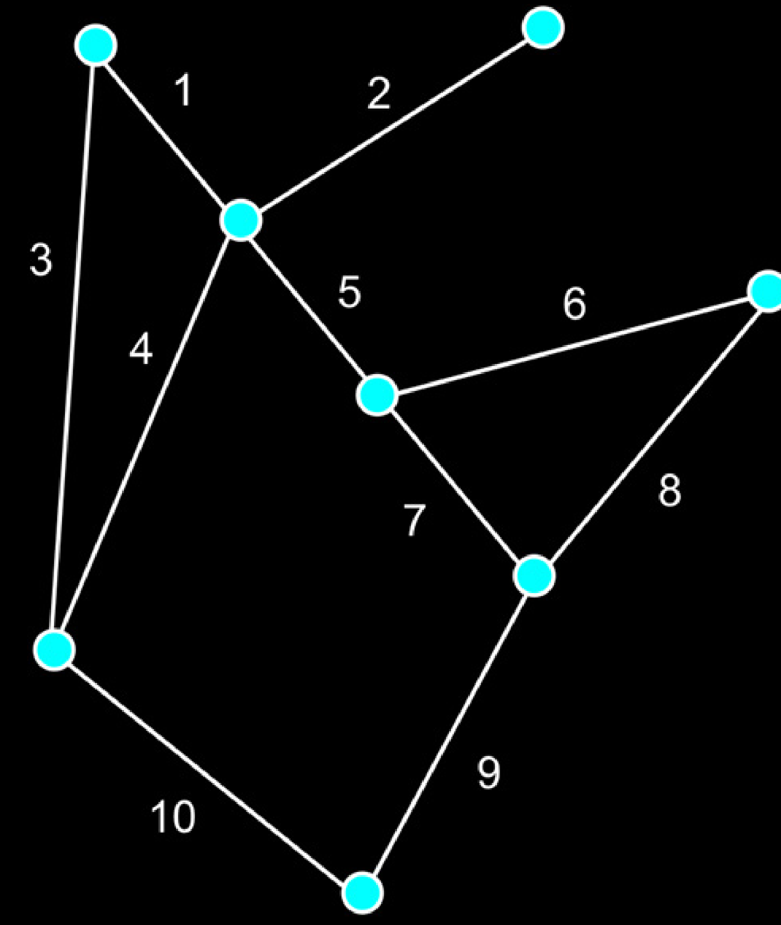
Topologic (geometric) distance
1 step

Example Graph

Vertices = 8



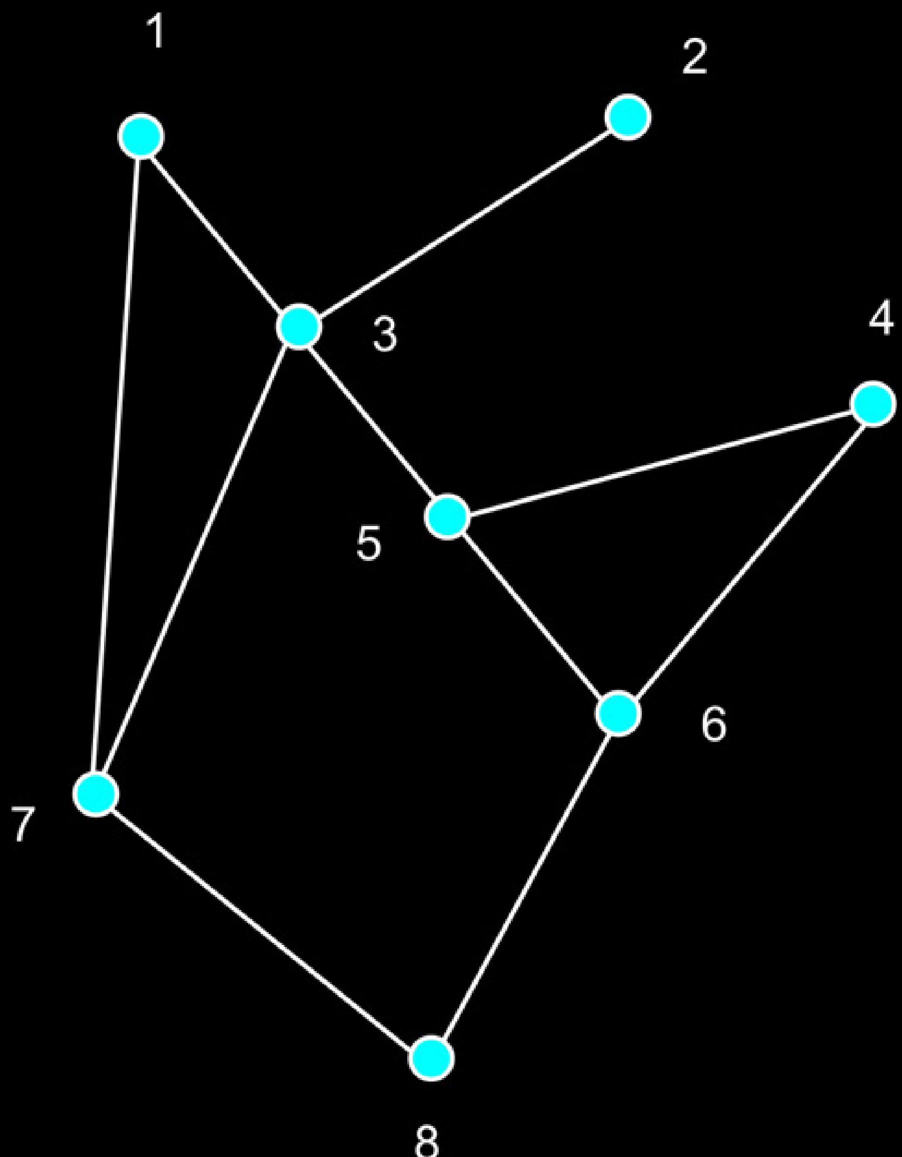
Edges = 10



Diameter – the number of links needed to connect the two most remote nodes in a network.

- The route used must be the shortest possible.
- Backtracks, loops, and detours are excluded.
- Sometimes referred to as the ‘longest, shortest path’.

In other words, the diameter is the maximum number of steps to connect any two points on a graph.



Most Distant Nodes:

8 and 2

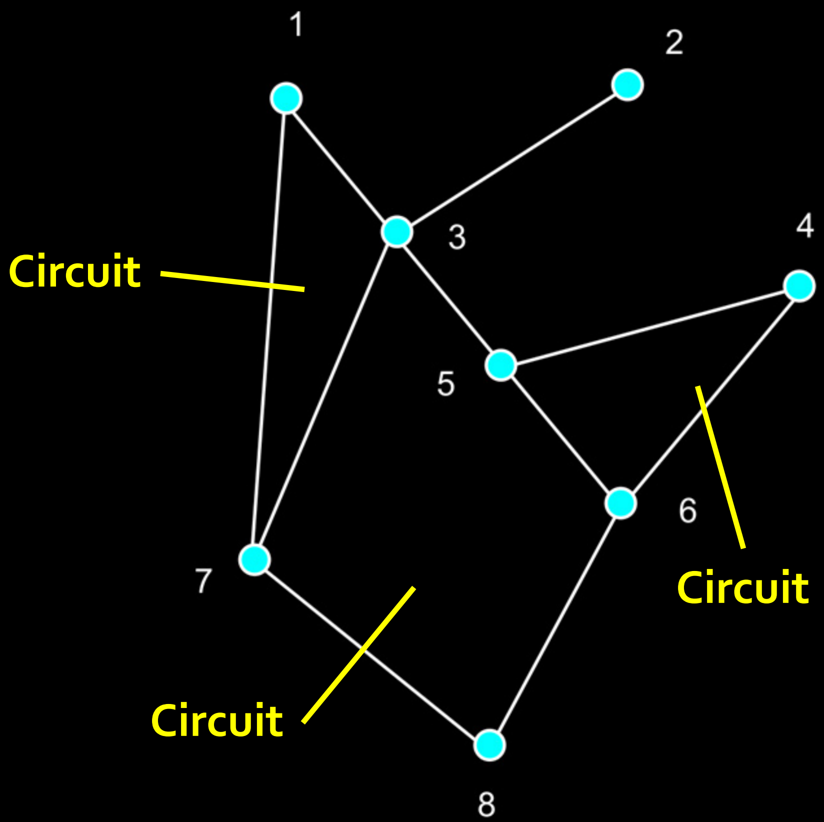
Longest Shortest Route

8 → 7 → 3 → 2

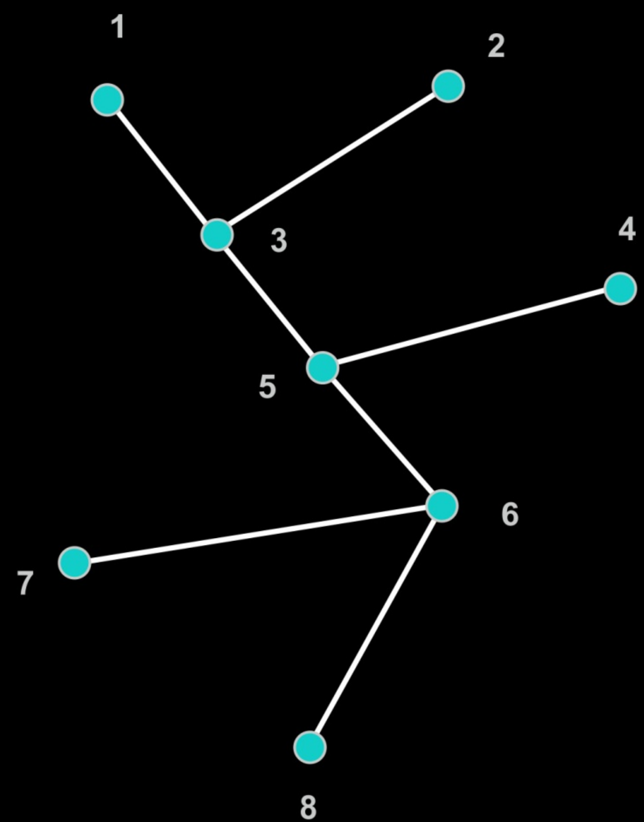
Diameter = 3

Tree **– a connected acyclic simple graph, which therefore has no complete circuits.**

Simple graph



Tree



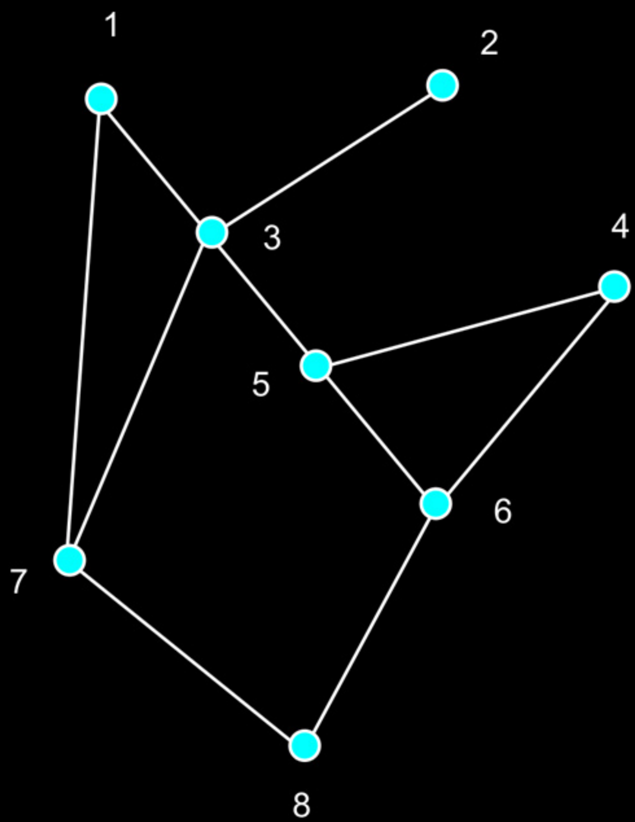
Cyclomatic Number – An index that is the difference between the number of edges and vertices.

$$\mu = e - v + p$$

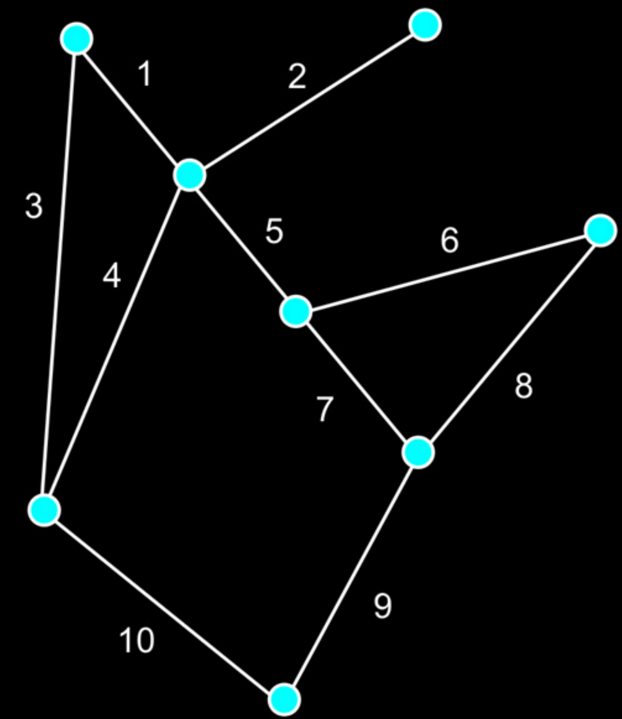
where μ is the cyclomatic number, e is the number of edges, v is the number of vertices, and p is the number of subgraphs.

Note: in most cases $p = 1$.

Vertices

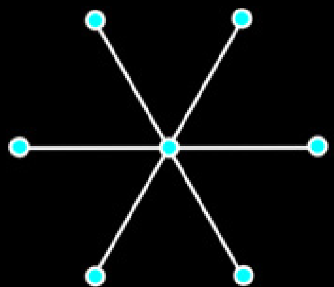


Edges



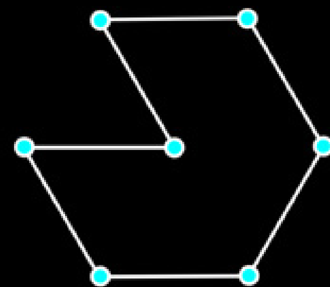
$$\mu = e - v + p$$

$$\mu = 10 - 8 + 1 = 3$$



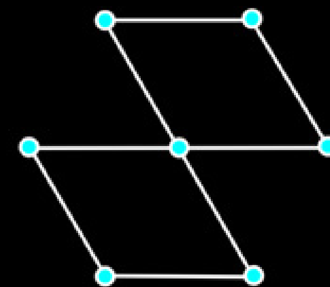
$e = 6$
$v = 7$
$p = 1$

$6 - 7 + 1 = 0$



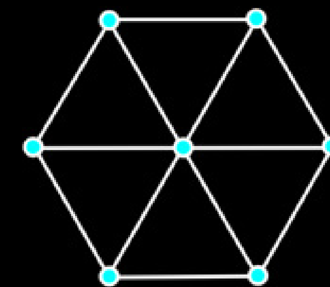
$e = \mathbf{7}$
$v = 7$
$p = 1$

$7 - 7 + 1 = 1$



$e = 8$
$v = 7$
$p = 1$

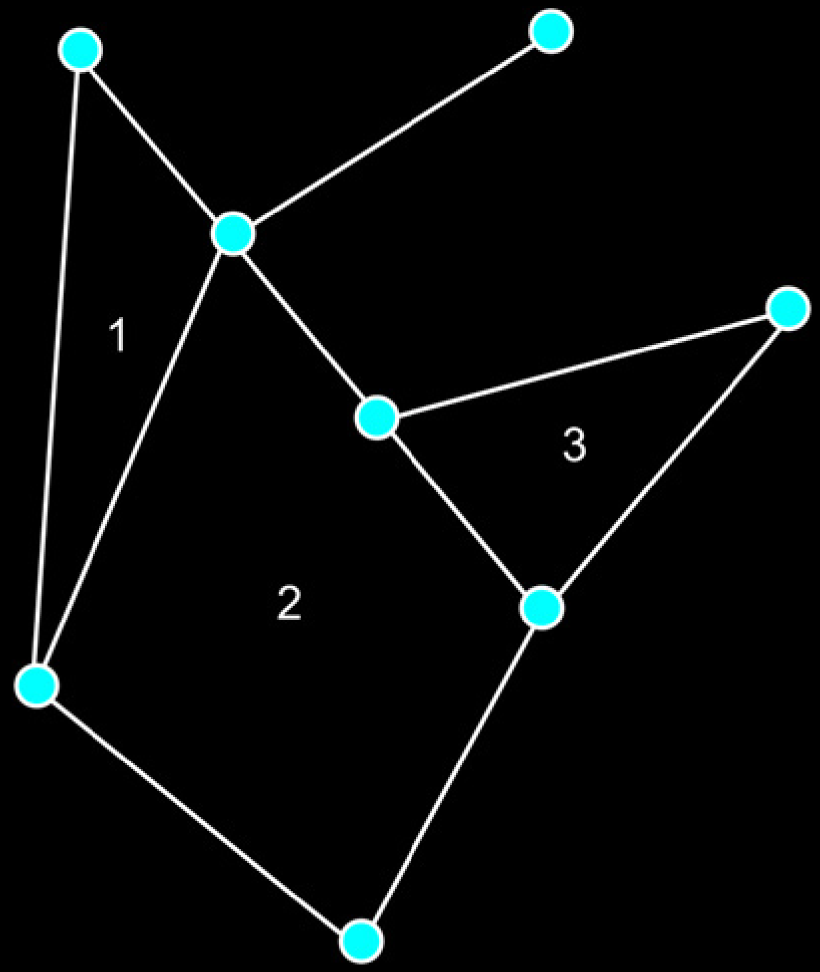
$8 - 7 + 1 = 2$

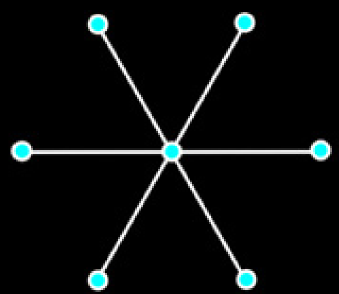


$e = 12$
$v = 7$
$p = 1$

$12 - 7 + 1 = 6$

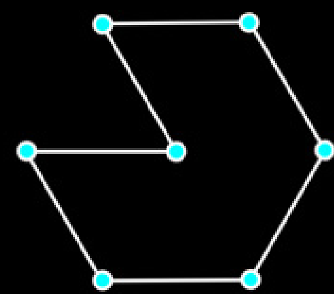
Therefore, the cyclomatic number is essentially the number of closed circuits in the graph. It is a measure of route redundancy.





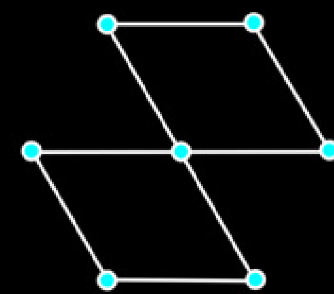
$e = 6$
$v = 7$
$p = 1$

$6 - 7 + 1 = 0$



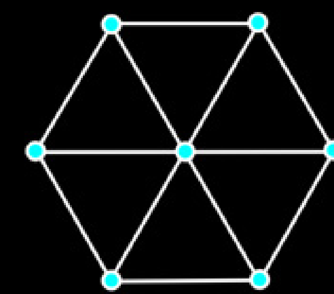
$e = \mathbf{7}$
$v = 7$
$p = 1$

$7 - 7 + 1 = 1$



$e = 8$
$v = 7$
$p = 1$

$8 - 7 + 1 = 2$

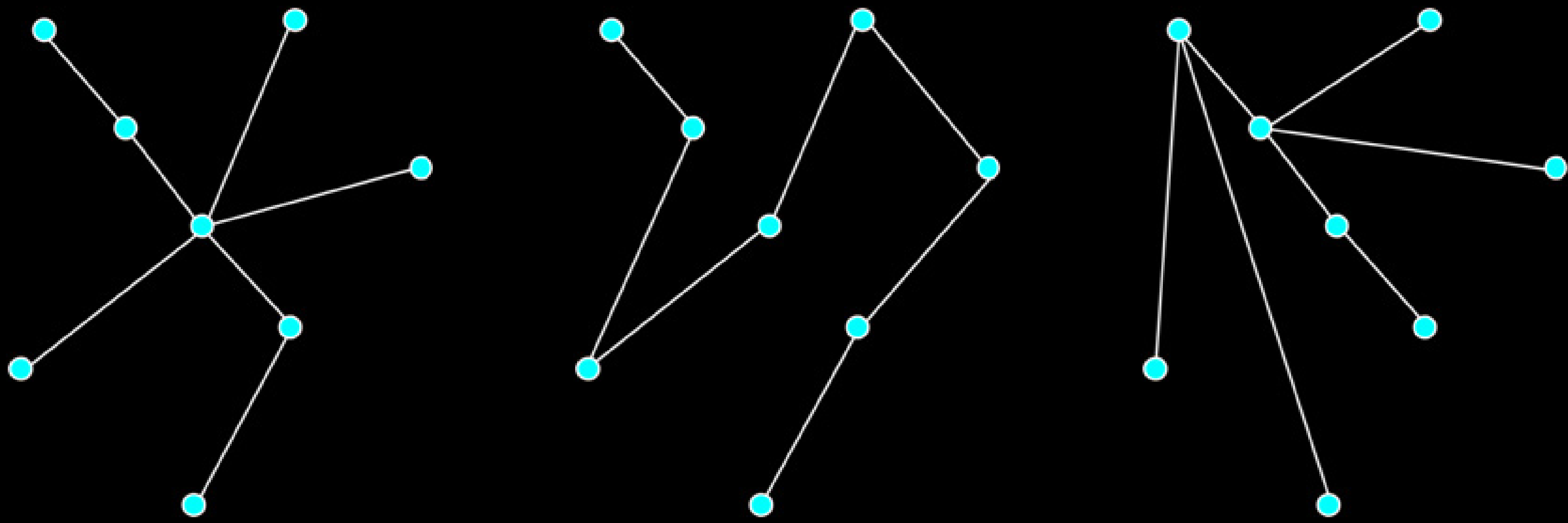


$e = 12$
$v = 7$
$p = 1$

$12 - 7 + 1 = 6$

Minimum number of edges in a graph:

$$Min\ \mu = v - 1$$



$$Min\ \mu = 8 - 1 = 7$$

Maximum number of circuits in a graph:

$$\text{Max circuits} = \tfrac{1}{2}v\,(v-1)-(v-1)$$

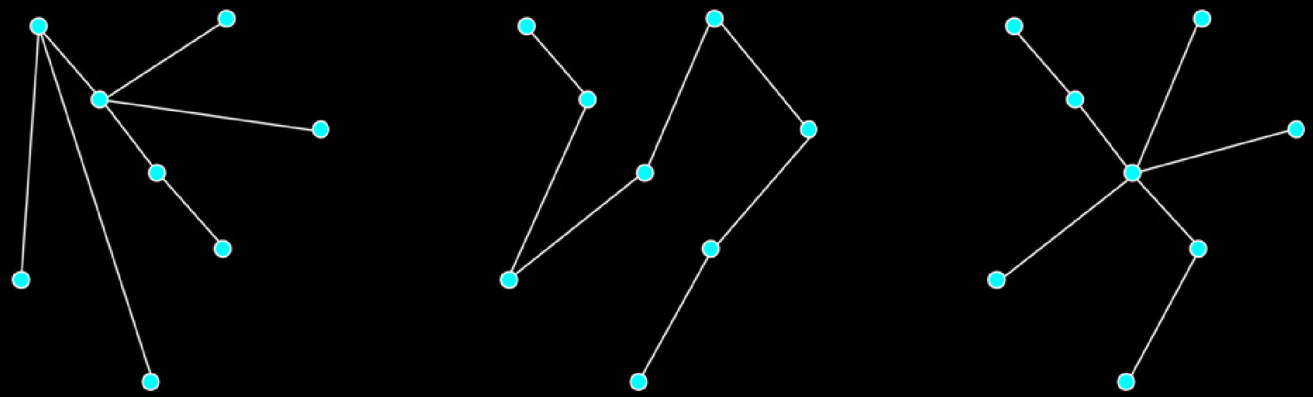
Note: not all vertices are shown.

$$\text{Max circuits} = \tfrac{1}{2}8\,(8-1)-(8-1)$$

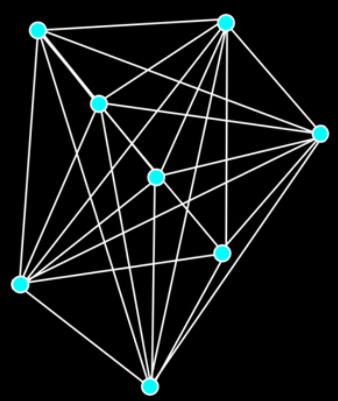
$$\text{Max circuits} = (4)(7)-7 = 21$$

Notice that there are several possible graphs that can be drawn base on *Min μ*, but there is only one possible *Max μ*.

Min edges



Max circuits



Gamma **– a measure of graph connectivity and is a simple measure to use. It can be thought of as *the percent existing routes to potential routes*.**

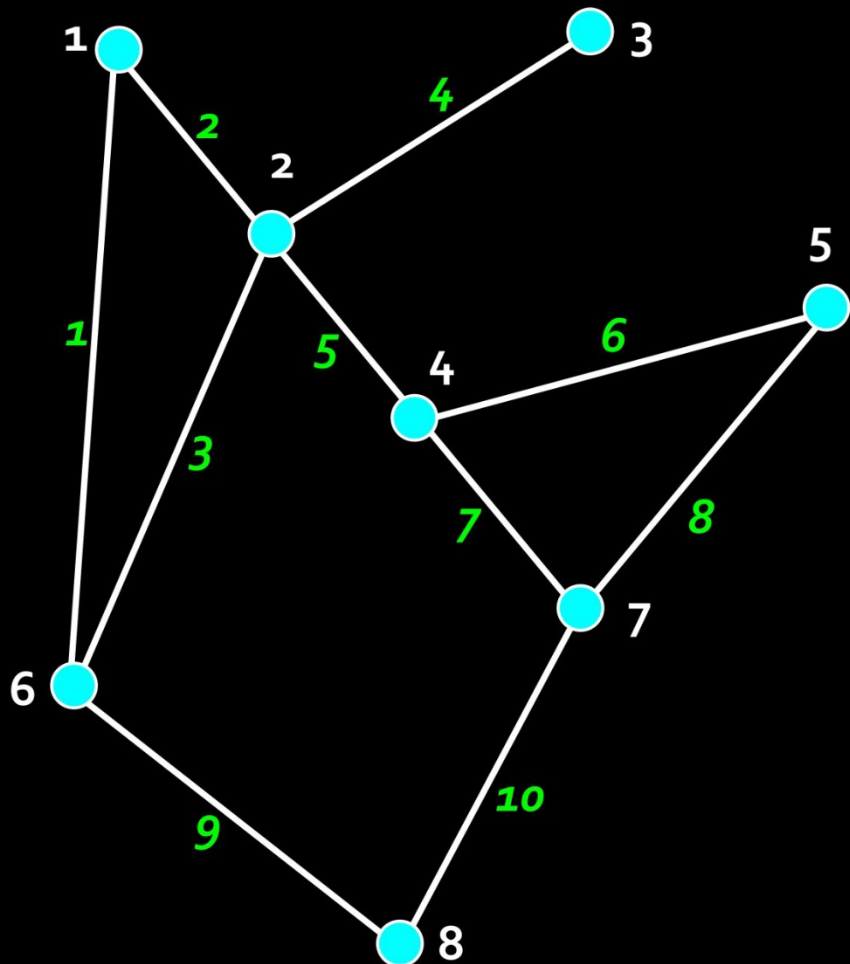
$$\gamma = e / (1/2v(v-1))$$

or

$$\gamma = e / (0.5v)(v-1)$$

$v = 8$

$e = 10$



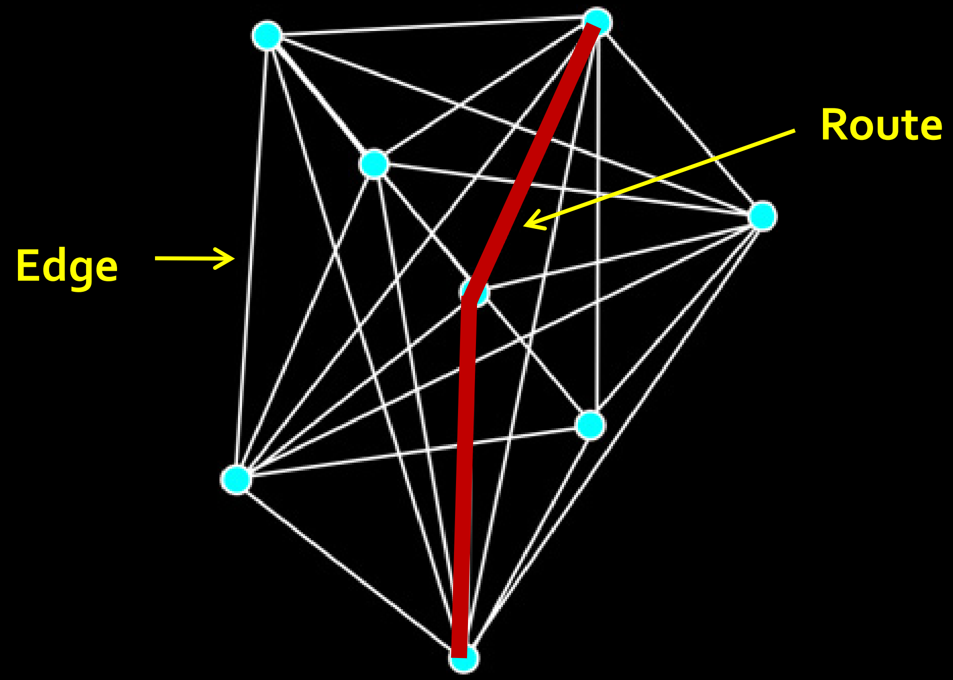
$\gamma = 10 / (0.5 \times 8)(8-1)$

$\gamma = 10 / (4)(7)$

$\gamma = 10 / 28$

$\gamma = 0.357$

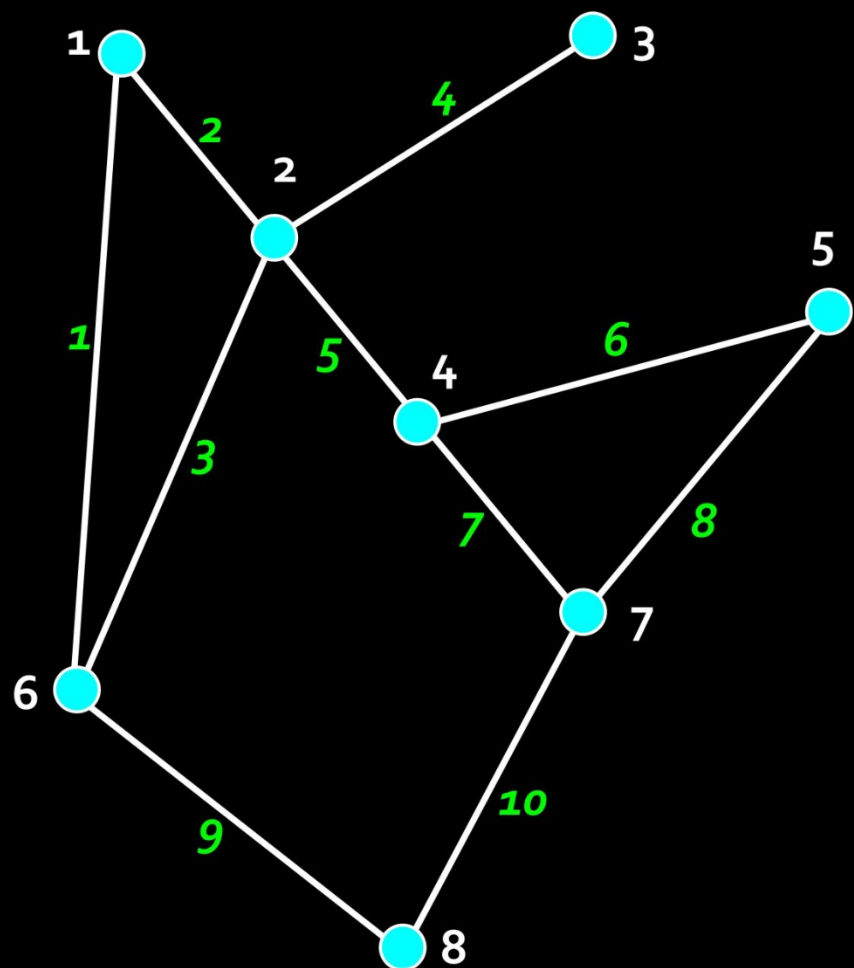
Remember that max μ calculates EDGES and that gamma is working with ROUTES.



***Beta* – a measure of graph connectivity that can be interpreted as *the average number of edges per vertex (average number of links per node)*.**

$$\beta = e / v$$

$v = 8$
$e = 10$



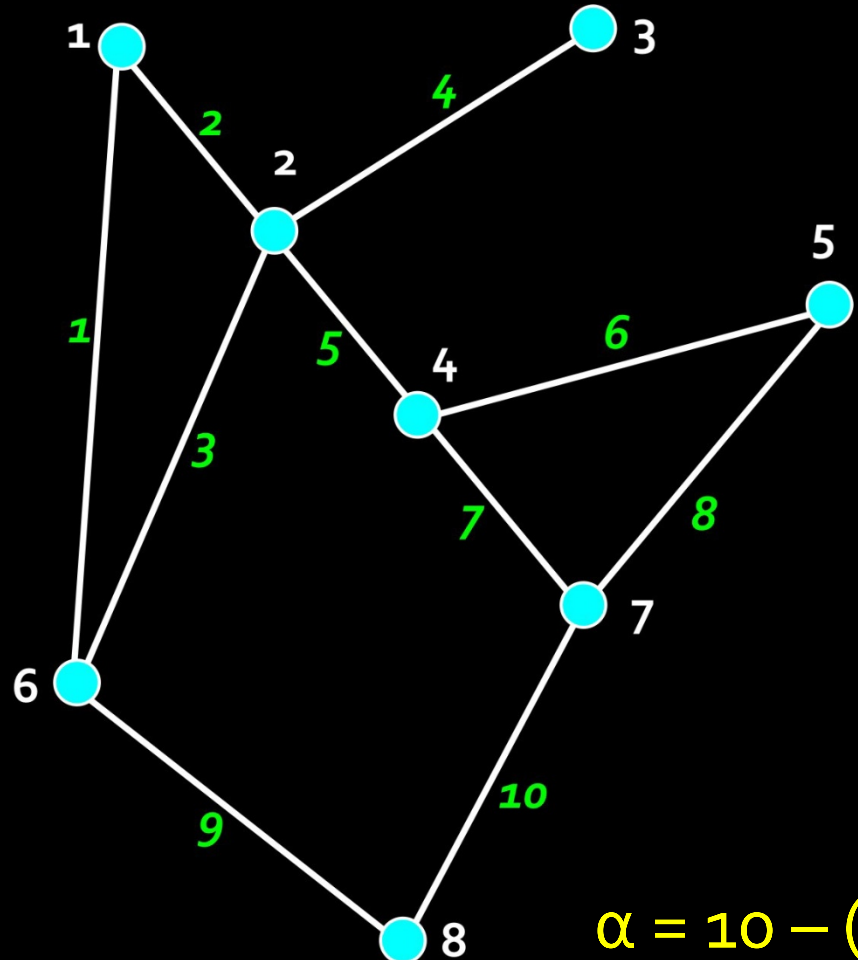
$\beta = 10 / 8$
$\beta = 1.25$

Alpha* – a measure of graph connectivity that can be interpreted as *the ratio of existing circuits to the maximum possible circuits.

$$\alpha = e - (v\text{-}1) \,/\, (0.5v(v\text{-}1)) - (v\text{-}1)$$

v = 8

e = 10



$\alpha = 10 - (8-1) / (0.5(8)(8-1)) - (8-1)$

$\alpha = 3 / (4)(7) - 7$

$\alpha = 3 / 21$

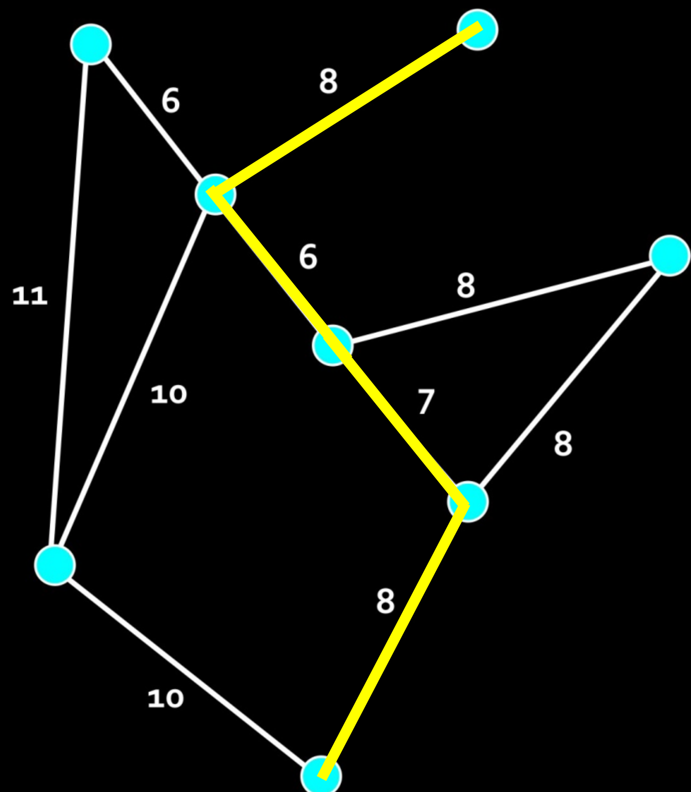
$\alpha = 0.143$

Pi Index – **a the relationship between the total length of the graph *L(G)* and the distance along its diameter *D(d)*. A high index shows a developed network.** ***It is a measure of distance per units of diameter and an indicator of the shape of the network.***

$$\pi = \frac{L(G)}{D(d)}$$

$L(G) = 82$

$D(d) = 29$



$$\pi = \frac{11+10+10+8+8+8+7+6+8+6}{8+7+6+8}$$

$$\pi = \frac{82}{29}$$

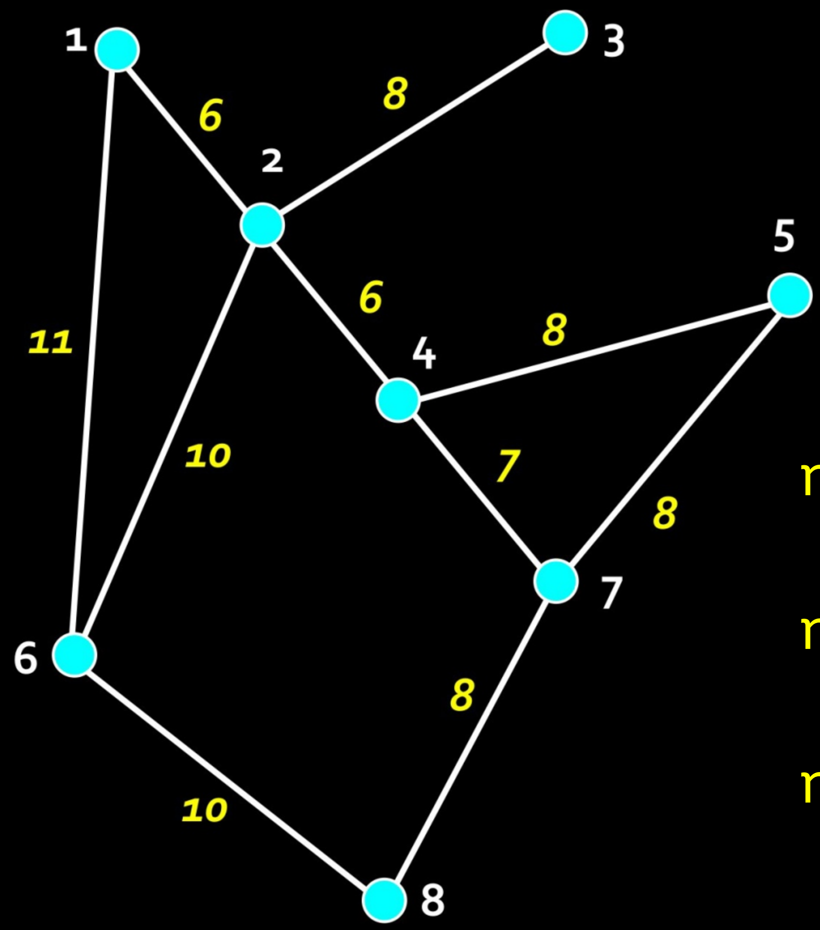
$$\pi = 2.83$$

Eta – the average length per link. **Adding new nodes will cause a decrease in eta as the average length per link declines.**

$$\eta = \frac{L(G)}{e}$$

L(G) = 82

e = 10



$$\eta = \frac{11+10+10+8+8+8+7+6+8+6}{10}$$

$$\eta = \frac{82}{10}$$

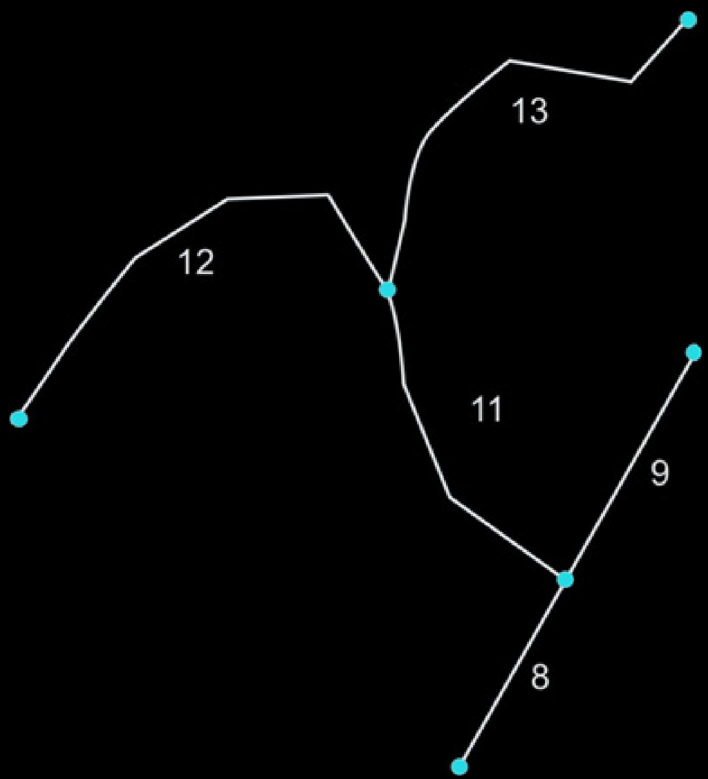
$$\eta = 8.2$$

Detour Index – a measure of the efficiency of a network in terms of how well it overcomes distance **or the friction of distance. The closer the detour index gets to 1, the more the network is spatially efficient. Networks having a detour index of 1 are rarely, if ever, seen and most networks would fit on an asymptotic curve getting close to 1, but never reaching it.**

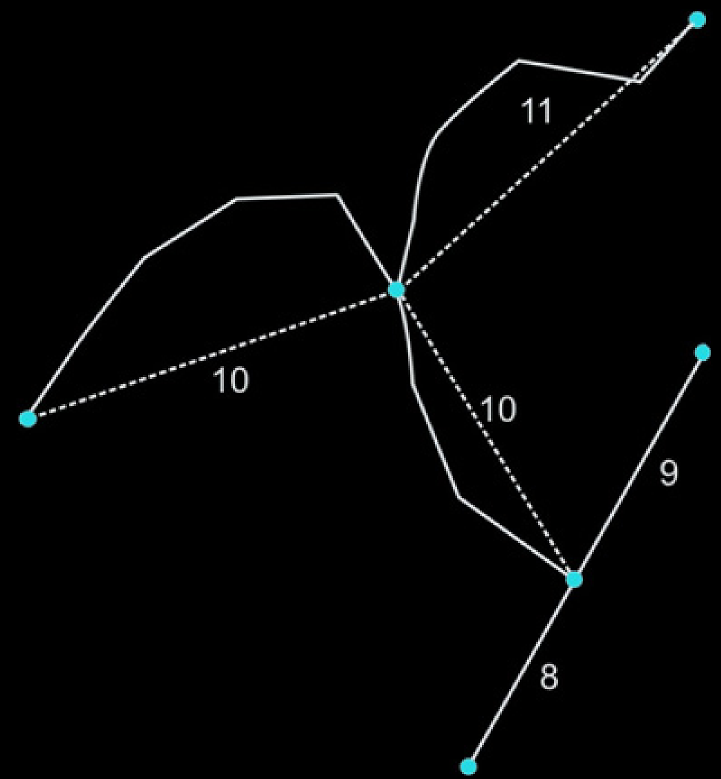
$$DI = \frac{DD}{TD}$$

The straight distance (*DD*) between two nodes may be 40 km but the transport distance (*TD*; real distance) is 50 km. The detour index is thus 0.8 (40 / 50).

Transport Distance (TD)



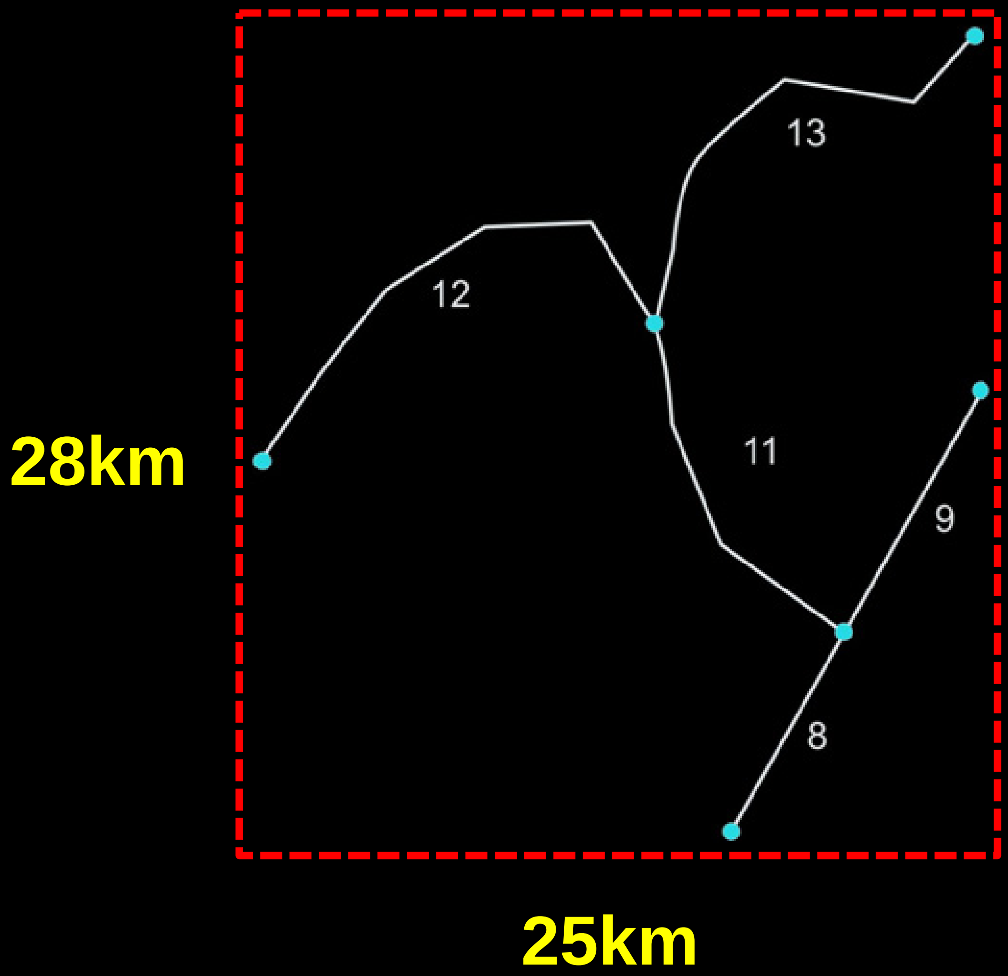
Straight Distance (DD)



$$DI = \frac{(10 + 11 + 10 + 8 + 9)}{(12 + 13 + 11 + 8 + 9)} = \frac{48}{53} = 0.91$$

Network Density – **measures the territorial occupation of a transport network in terms of km *of links (L) per square kilometers of surface (S)*. The higher it is, the more a network is developed.**

$$ND = \frac{L}{S}$$



$$ND = \frac{(12 + 13 + 11 + 8 + 9)}{(25 \times 27)}$$

$$ND = \frac{53}{100}$$

$$ND = 0.076\ \text{km/km}^2$$