Transportation Network Analysis
Connectivity Index

Connectivity – the relative degree of connectedness within a transportation network.

- High connectivity = low isolation, high accessibility.
- Low connectivity = high isolation, low accessibility.

Connectivity is a measure of accessibility *without* regard to distance.

- Places with high connectivity are often considered *important* since they are the best connected.

Connectivity Matrix

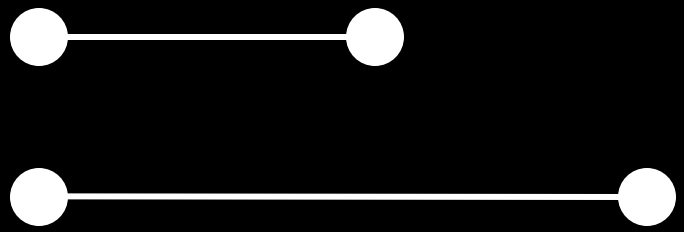
First must reduce the transportation network to a matrix consisting of ones (1) and zeros (0).

- If two locations (vertices) are directly connected by a link (edge), code with a 1.

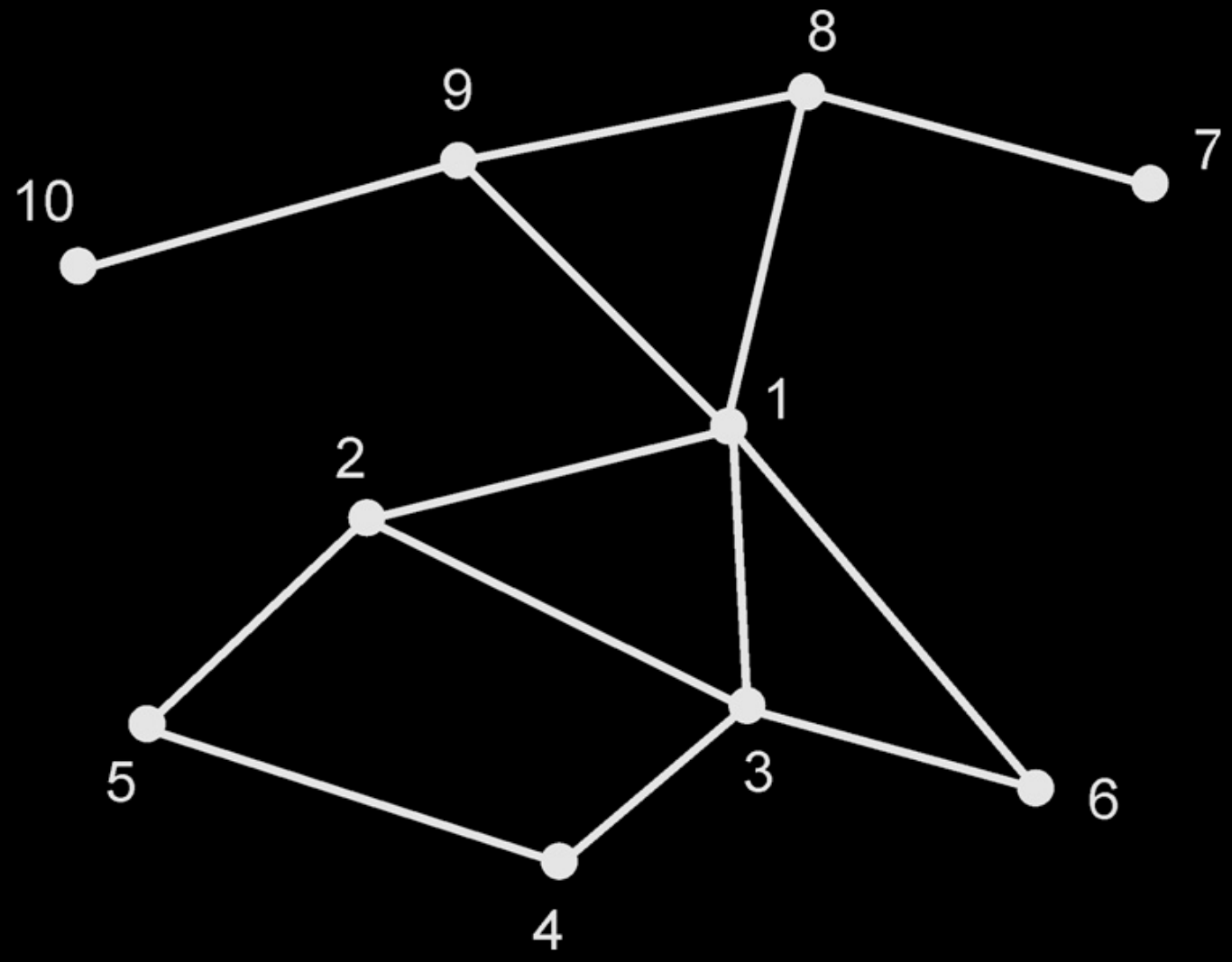
- If two locations (vertices) are NOT directly connected by a link (edge), code with a 0.

Connectivity is based on topologic distance.

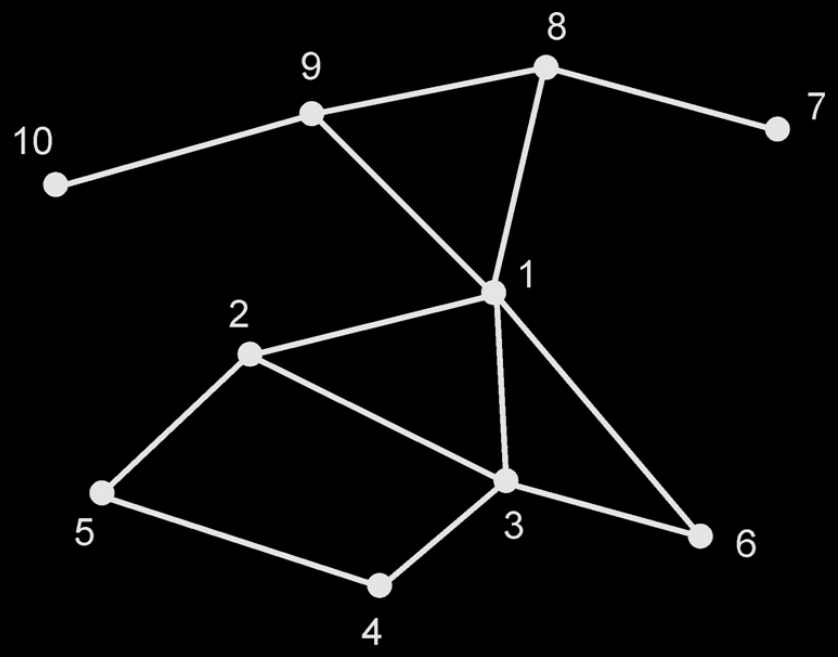
Topological distance – the number of direct connections or steps separating two nodes.



Both of these have a topological distance of 1.

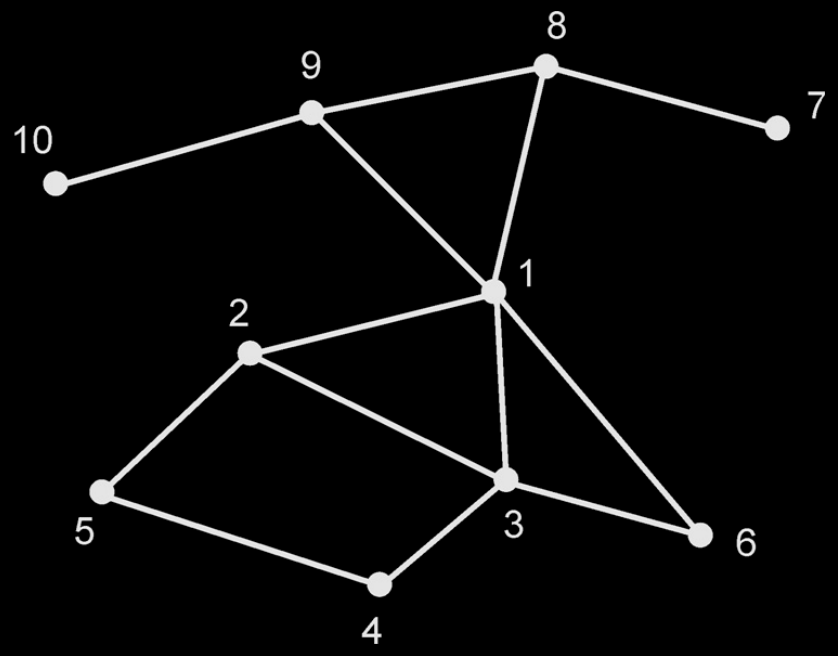

8
9
7
10
1
2
5
3
6
4

Step 1: Number the vertices and create a matrix where rows = *v* and columns = *v*.



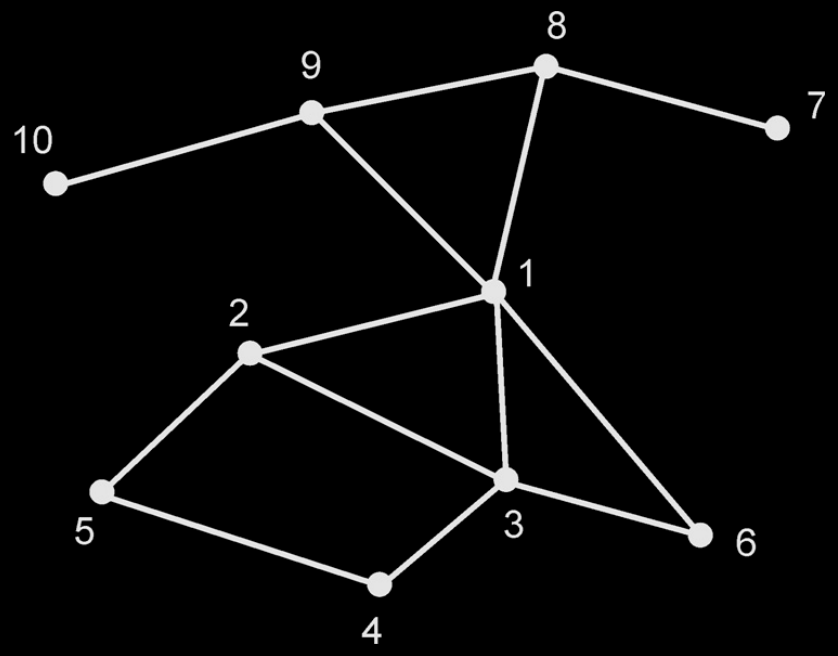
	1	2	3	4	5	6	7	8	9	10
1	0									
2		0								
3			0							
4				0						
5					0					
6						0				
7							0			
8								0		
9									0	
10										0

Step 2: Code direct connections as 1. Use the lower half of the matrix.



	1	2	3	4	5	6	7	8	9	10
1	0									
2	1	0								
3	1	1	0							
4			1	0						
5		1		1	0					
6	1		1			0				
7							0			
8	1						1	0		
9	1							1	0	
10									1	0

Step 3: Transpose (copy) the lower half of the matrix to the upper half of the matrix.

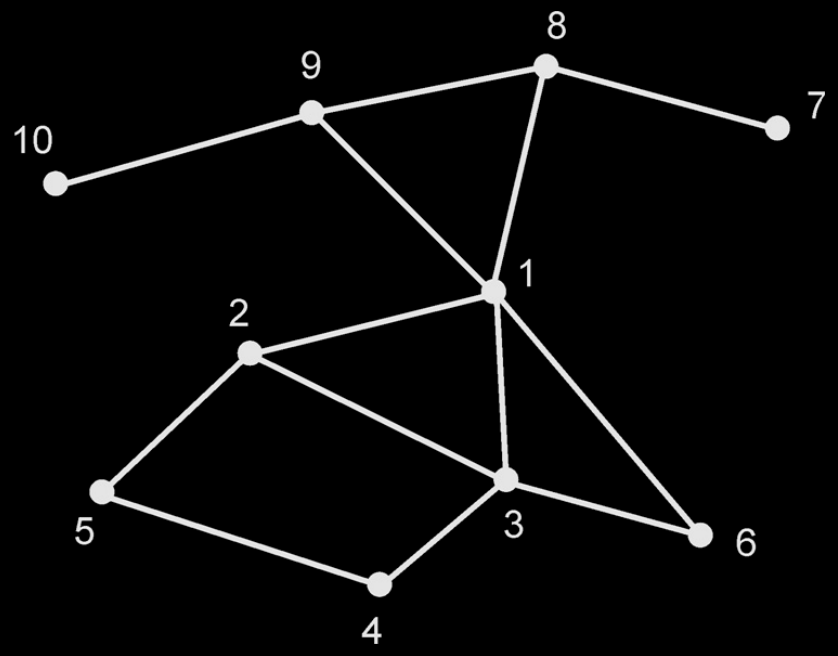


	1	2	3	4	5	6	7	8	9	10
1	0	1	1			1		1	1	
2	1	0	1		1					
3	1	1	0	1		1				
4			1	0	1					
5		1		1	0					
6	1		1			0				
7							0	1		
8	1						1	0	1	
9	1							1	0	1
10									1	0

Transposing the matrix is done to account for flow in both directions.

- **A → B and B → A**

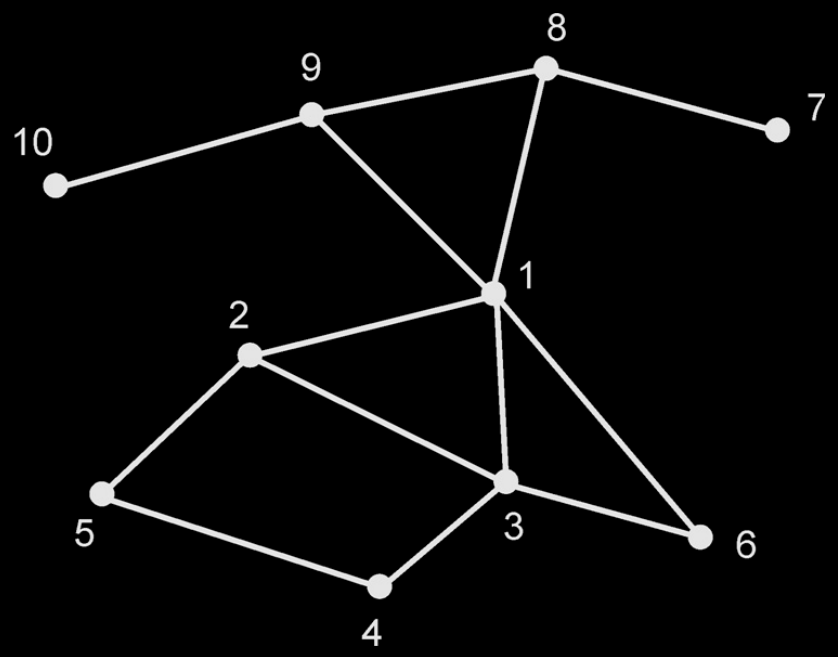
Step 4: Code all other non-directly connected dyads (cells) with a 0.



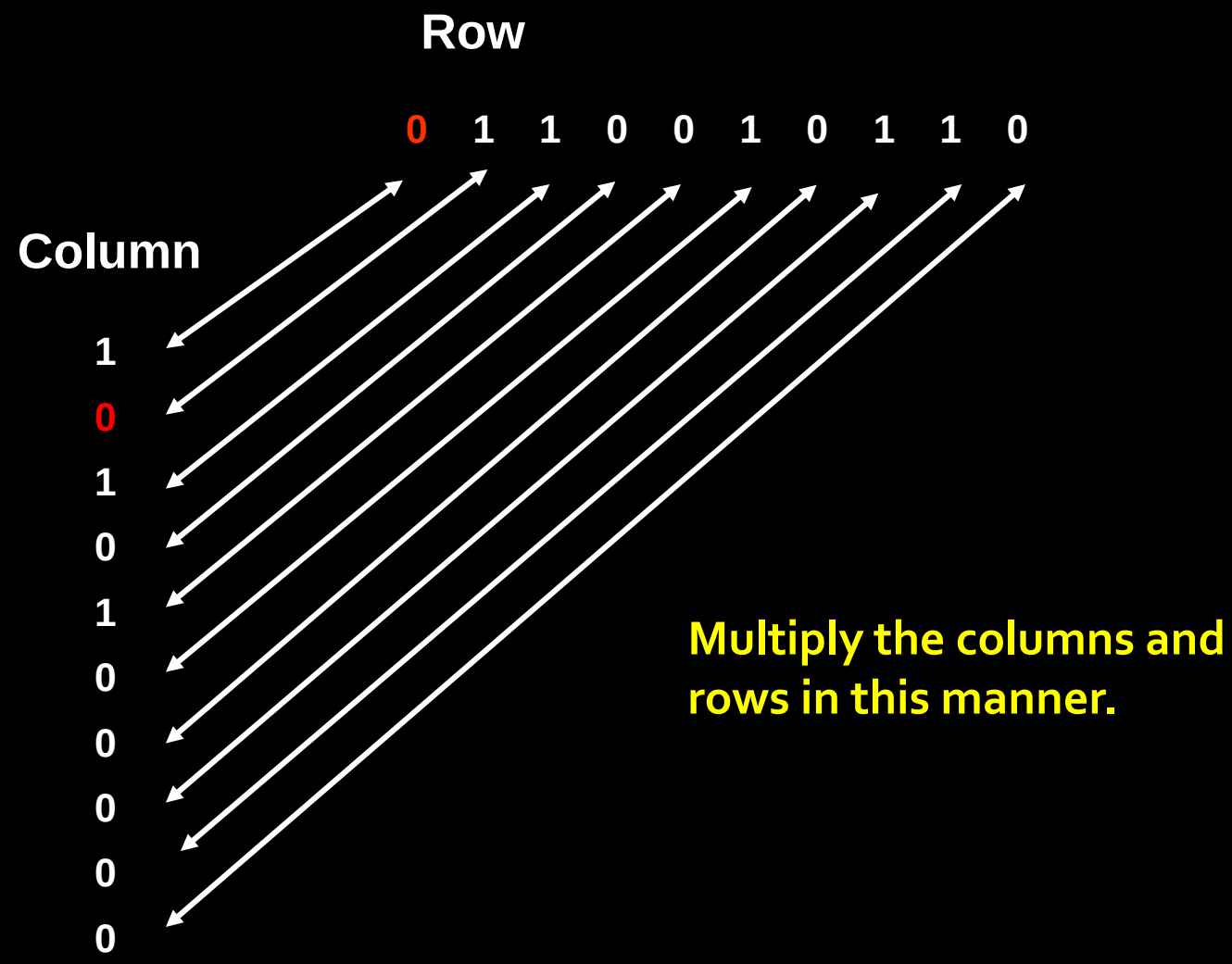
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	0	1	1	0	0	1	0	1	1	0
2	1	0	1	0	1	0	0	0	0	0
3	1	1	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0	0	0
5	0	1	0	1	0	0	0	0	0	0
6	1	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	1	0	0	0	0	0	1	0	1	0
9	1	0	0	0	0	0	0	1	0	1
10	0	0	0	0	0	0	0	0	1	0

Step 5: Power the matrix (multiply the matrix by itself) to determine all 2-step linkages.

Example: Row 1, Column 2



	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	1	0	1	1	0
2	1	0	1	0	1	0	0	0	0	0
3	1	1	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0	0	0
5	0	1	0	1	0	0	0	0	0	0
6	1	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	1	0	0	0	0	0	1	0	1	0
9	1	0	0	0	0	0	0	1	0	1
10	0	0	0	0	0	0	0	0	1	0

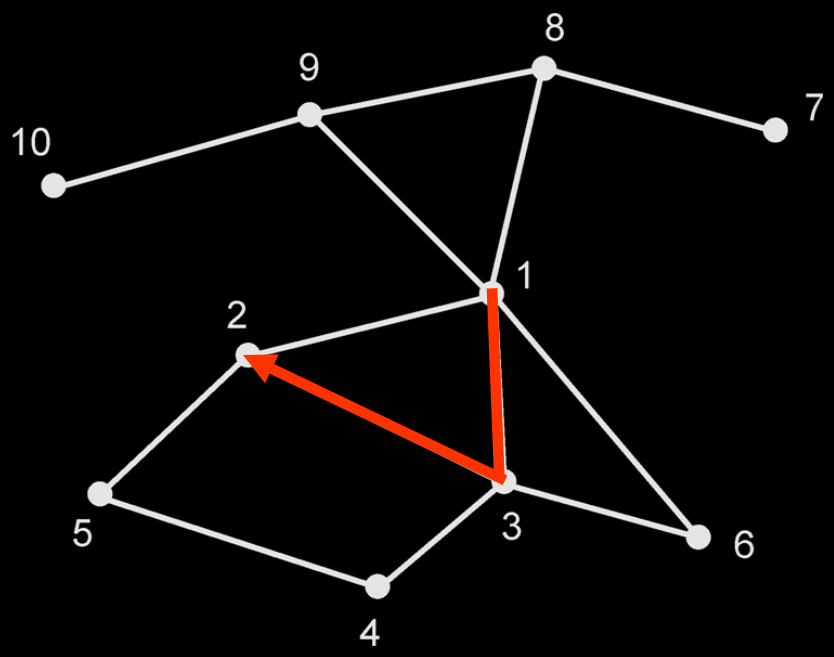

Row
0 1 1 0 0 1 0 1 1 0
Column
1
0
1
0
1
1
0
0
0
0
0
Multiply the columns and rows in this manner.

0
0
1
0
0
0
0
0
0
0

Sum = 1

Then sum the column.

There is only ONE two-step route between vertices 1 and 2.



	1	2	3	4	5	6	7	8	9	10
1		1								
2										
3										
4										
5										
6										
7										
8										
9										
10										

This process is done for every dyad (cell) in the matrix.

The resulting matrix represents all possible 2-step combinations.

	1	2	3	4	5	6	7	8	9	10
1	5	1	2	1	1	1	1	1	1	1
2	1	3	1	2	0	2	0	1	1	0
3	2	1	4	0	2	1	0	1	1	0
4	1	2	0	2	0	1	0	0	0	0
5	1	0	2	0	2	0	0	0	0	0
6	1	2	1	1	0	2	0	1	1	0
7	1	0	0	0	0	0	1	0	1	0
8	1	1	1	0	0	1	0	3	1	1
9	1	1	1	0	0	1	1	1	3	0
10	1	0	0	0	0	0	0	1	0	1

The powered and original matrix are then added.

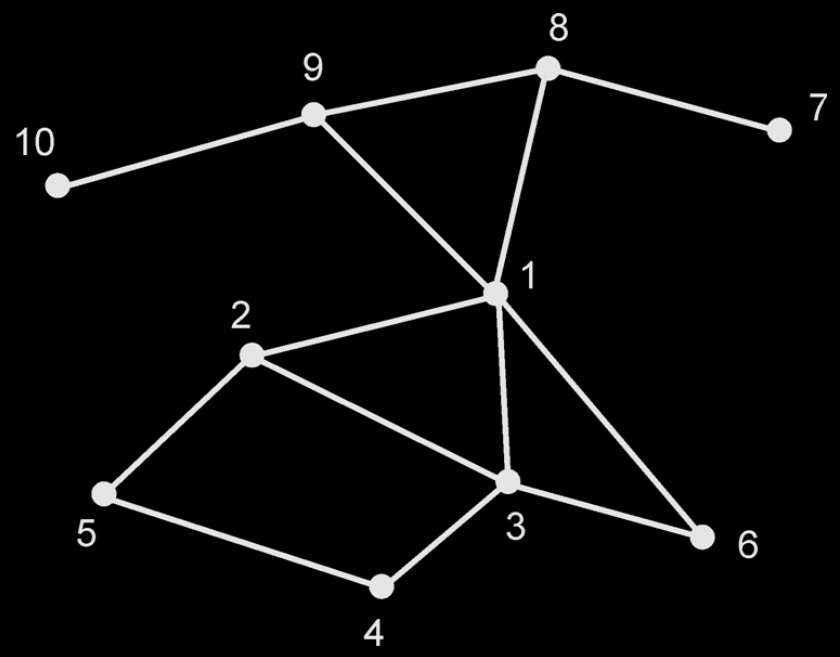
C^1 Matrix (Original)

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	1	0	1	1	0
2	1	0	1	0	1	0	0	0	0	0
3	1	1	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0	0	0
5	0	1	0	1	0	0	0	0	0	0
6	1	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	1	0	0	0	0	0	1	0	1	0
9	1	0	0	0	0	0	0	1	0	1
10	0	0	0	0	0	0	0	0	1	0

C^2 Matrix

	1	2	3	4	5	6	7	8	9	10
1	5	1	2	1	1	1	1	1	1	1
2	1	3	1	2	0	2	0	1	1	0
3	2	1	4	0	2	1	0	1	1	0
4	1	2	0	2	0	1	0	0	0	0
5	1	0	2	0	2	0	0	0	0	0
6	1	2	1	1	0	2	0	1	1	0
7	1	0	0	0	0	0	1	0	1	0
8	1	1	1	0	0	1	0	3	1	1
9	1	1	1	0	0	1	1	1	3	0
10	1	0	0	0	0	0	0	1	0	1

This matrix now represents all ONE and TWO step routes. The new matrix is then powered again and the whole process is repeated.



	1	2	3	4	5	6	7	8	9	10
1	5	2	3	1	1	2	1	2	2	1
2	1	3	2	2	1	2	0	1	1	0
3	3	2	4	1	2	2	0	1	1	0
4	1	2	1	2	1	1	0	0	0	0
5	1	1	2	1	2	0	0	0	0	0
6	2	2	2	1	0	2	0	1	1	0
7	1	0	0	0	0	0	1	1	1	0
8	2	1	1	0	0	1	1	3	2	1
9	2	1	1	0	0	1	1	2	3	1
10	1	0	0	0	0	0	0	1	1	1

- The diagonal represents all routes from a vertex back to itself.

- *Matrix redundancy* – accounting for all routes with a matrix. Tends to inflate connectivity results and favors central positions.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	5	2	3	1	1	2	1	2	2	1
2	1	3	2	2	1	2	0	1	1	0
3	3	2	4	1	2	2	0	1	1	0
4	1	2	1	2	1	1	0	0	0	0
5	1	1	2	1	2	0	0	0	0	0
6	2	2	2	1	0	2	0	1	1	0
7	1	0	0	0	0	0	1	1	1	0
8	2	1	1	0	0	1	1	3	2	1
9	2	1	1	0	0	1	1	2	3	1
10	1	0	0	0	0	0	0	1	1	1

The powering and adding procedures continue until ALL zero (0) cells are filled.

The number of powering procedures needed to do this is termed the *diameter*.

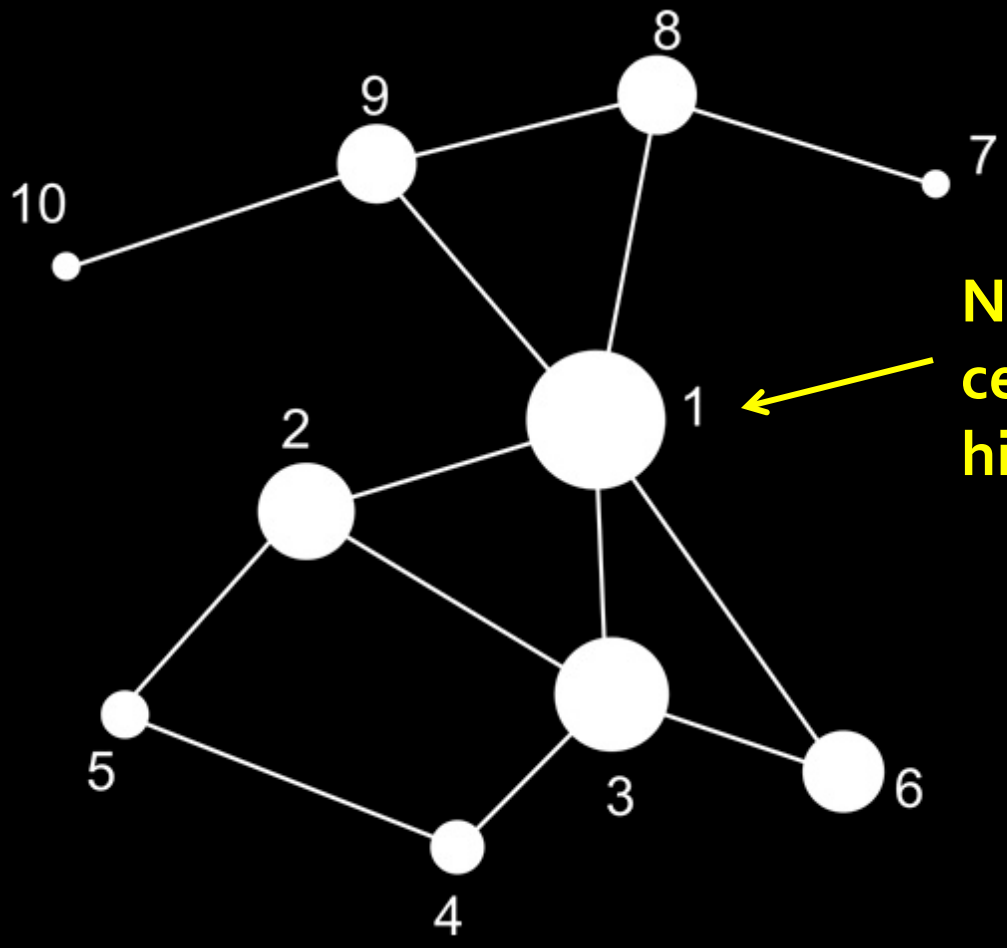
***Diameter* – the fewest number of step needed to connect the vertices which are the farthest apart topologically.**

- **Backtracking, detours, or loops are excluded.**

The final matrix is termed the total connectivity matrix. Summing the rows gives the total connectivity for each vertex.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*Total*
1	48	26	35	14	14	23	9	23	23	9	224
2	26	26	23	16	9	20	3	14	14	3	154
3	35	23	36	12	17	20	4	16	16	4	183
4	14	16	12	12	6	11	1	5	5	1	83
5	14	9	17	6	11	6	1	4	4	1	73
6	23	20	20	11	6	17	3	13	13	3	129
7	9	3	4	1	1	3	4	6	7	2	40
8	23	14	16	5	4	13	6	20	17	7	125
9	23	14	16	5	4	13	7	17	20	6	125
10	9	3	4	1	1	3	2	7	6	4	40

The total connectivity values can then be mapped to help determine theoretical isolation or accessibility levels.



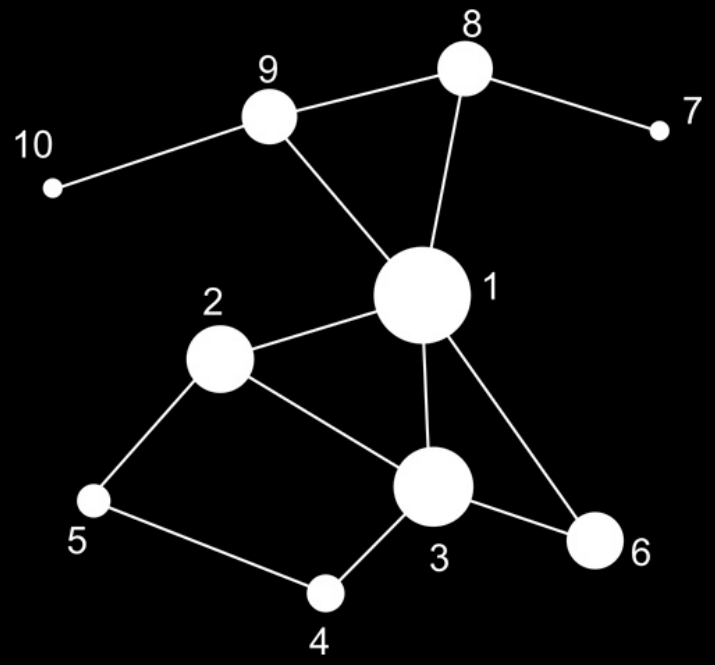
Note how the more central places have higher connectivity.

Therefore, connectivity is not only a measure of relative isolation, but also of *centrality*.

- Higher connectivity locations are more centrally located.
- Remember that the centrality is in terms of topologic and not real-world distance.

Network Measurements:

Vertices:	10
Edges:	13
Diameter:	4
Cyclomatic number:	4
Alpha:	0.11
Beta:	1.3
Gamma:	54.2



Connectivity Matrix Results:

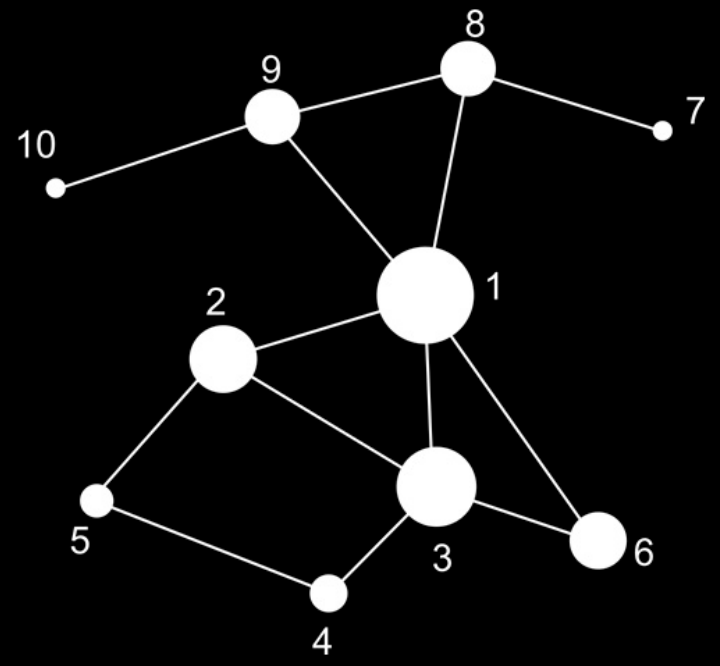
The least accessible node is 7 with 40 connections.

The most accessible node is 1 with 224 connections.

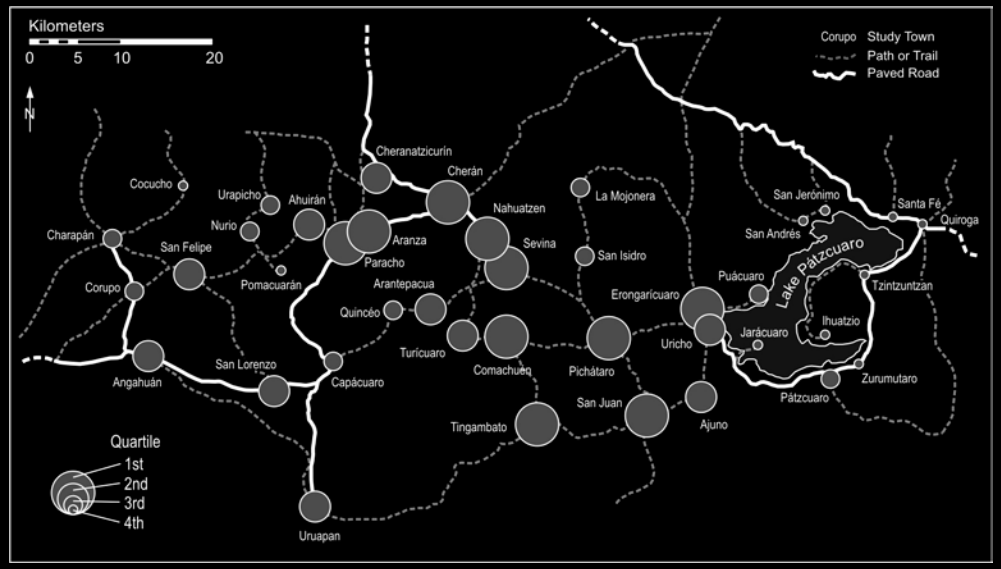
So what do these numbers tell us about this hypothetical network?

Diameter:	4	Takes 4 steps to connect all places.
Cyclomatic number:	4	There are 4 'extra' routes or 4 circuits.
Alpha:	0.11	There are 11% of all possible circuits.
Beta:	1.3	There are 1.3 roads per place.
Gamma:	0.542	There are 54.2% of the possible routes.

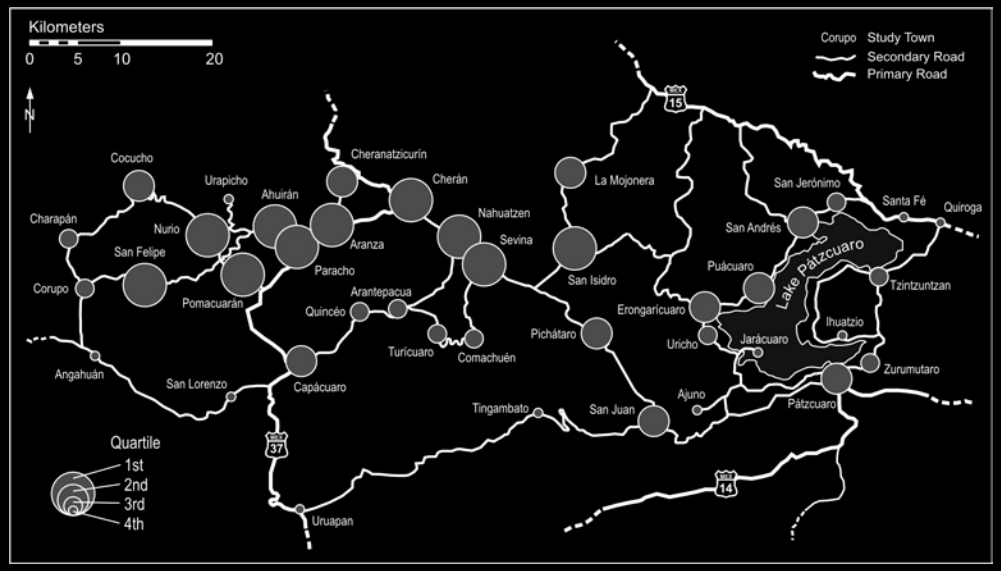
This network is about half way to being maximally connected. It is relatively well connected and location 1 is the most central.

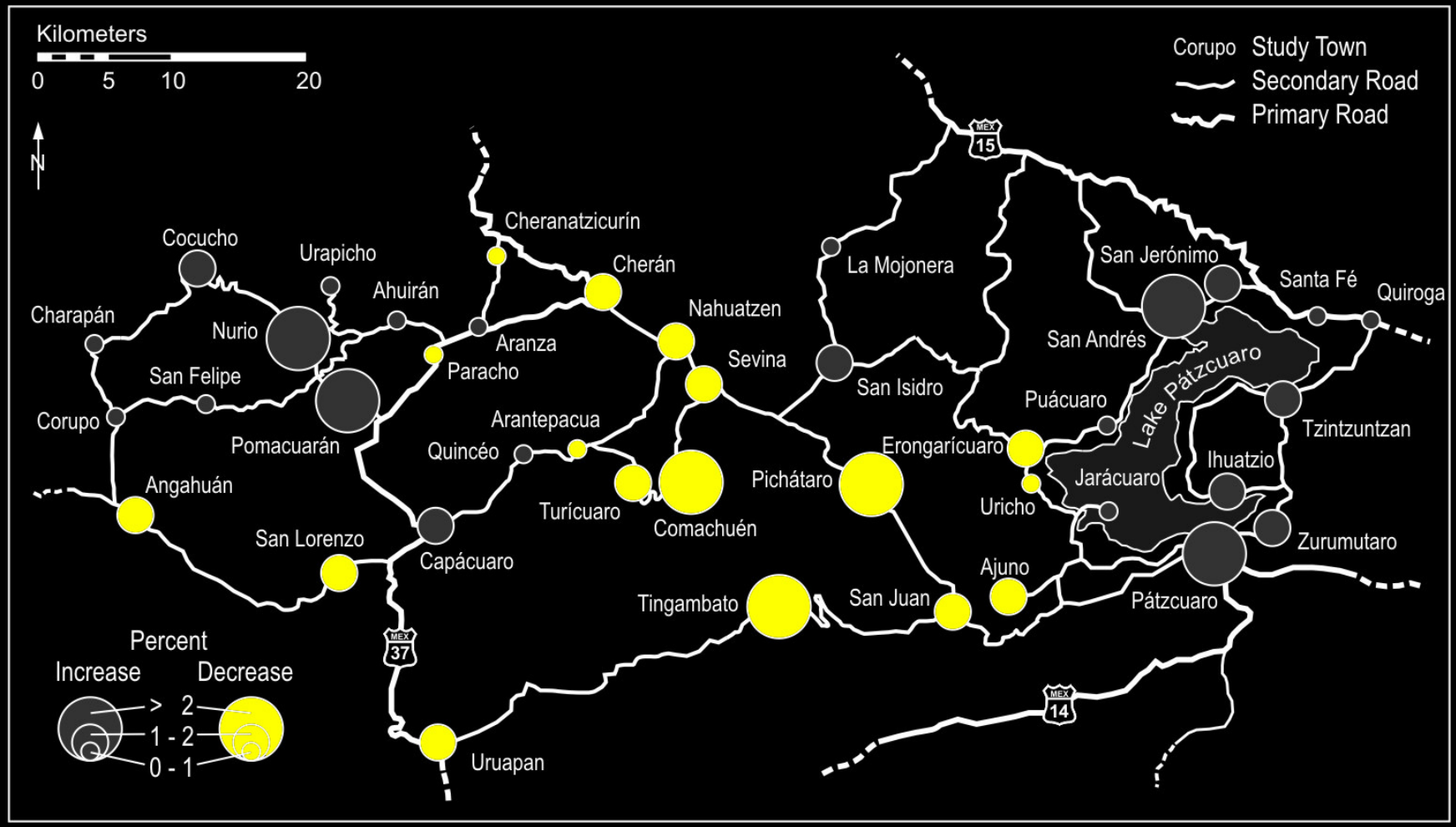


1940 connectivity



2000 connectivity




Kilometers
0 5 10 20
N
Corupo Study Town
Secondary Road
Primary Road
MEX 15
Cheranatzicurín
Cocucho
Urapicho
Cherán
Ahuirán
La Mojonera
San Jerónimo
Santa Fé
Quiroga
Charapán
Nurio
Nahuatzen
Aranza
Paracho
Sevina
San Andrés
Lake Pátzcuaro
San Felipe
San Isidro
Puácuaro
Corupo
Arantepacua
Tzintzuntzan
Pomacuarán
Quincéo
Erongarícuaro
Ihuatzio
Angahuán
Pichátaro
Jarácuaro
Turícuaro
Comachuén
Uricho
San Lorenzo
Capácuaro
Zurumutaro
Ajuno
Tingambato
San Juan
Pátzcuaro
MEX 37
MEX 14
Uruapan
Percent
Increase
Decrease
> 2
1 - 2
0 - 1