## 1.13 Homework Solutions

(13.A) Suppose $f(z) = \frac{z}{z+4}$. If $f(x+iy) = u(x,y) + iv(x,y)$, find $u$ and $v$.

$$\begin{aligned}
\frac{z}{z+4} &= \frac{x+iy}{x+4+iy} \\
&= \frac{x+iy}{x+4+iy}\left(\frac{x+4-iy}{x+4-iy}\right) \\
&= \frac{x^2+4x+y^2+i(-xy+xy+4y)}{(x+4)^2+y^2} \\
&= \frac{x^2+4x+y^2}{(x+4)^2+y^2} + i\frac{4y}{(x+4)^2+y^2}
\end{aligned}$$

So $u(x,y) = \frac{x^2+4x+y^2}{(x+4)^2+y^2}$ and $v(x,y) = \frac{4y}{(x+4)^2+y^2}$.

(3a) The radius is 1, which stays the same when squared. Squaring doubles the angle, so instead of 0 to $\frac{\pi}{4}$ you get 0 to $\frac{\pi}{2}$. The result is the upper right quarter on the closed unit disk.

(3b) Similar to (3a), except the pie piece goes from $\theta = 0$ to $\theta = \frac{3\pi}{4}$.

(3c) Similar to (3a), except you get the upper half circle of the unit disk.