2.17 Homework Solutions - Limits with Infinity

(10a)

$$\begin{aligned}
\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} &= \lim_{z\to 0} \frac{\frac{4}{z^2}}{(\frac{1}{z}-1)^2} \\
&= \lim_{z\to 0} \frac{\frac{4}{z^2}}{\frac{1}{z^2}-\frac{2}{z}+1} \\
&= \lim_{z\to 0} \frac{4}{1-2z+z^2} \\
&= \frac{4}{1} = 4
\end{aligned}$$

(10b)

$$\begin{aligned}
\lim_{z\to 1} \frac{(z-1)^3}{1} &= \frac{0}{1} = 0 \qquad SO \\
\lim_{z\to 1} \frac{1}{(z-1)^3} &= \infty
\end{aligned}$$

(10c)

$$\begin{aligned}
\lim_{z\to\infty} \frac{z-1}{z^2+1} &= \lim_{z\to 0} \frac{\frac{1}{z}-1}{\frac{1}{z^2}+1} \\
&= \lim_{z\to 0} \frac{z-z^2}{1+z^2} \\
&= \frac{0-0}{1+0} = 0 \qquad SO \\
\lim_{z\to\infty} \frac{z^2+1}{z-1} &= \infty
\end{aligned}$$