# Problems - Section 26

(1) Show that $u$ is harmonic and find a harmonic conjugate.

(1a)
$$u = 2x(1 - y)$$

(1b)
$$u = 2x - x^3 + 3xy^2$$

(4) Show that $v$ is a harmonic conjugate of $u$ if and only if $-u$ is a harmonic conjugate of $v$. (Hint: If $f(z)$ is analytic, then $-if(z)$ is also analytic.)

(5) Use the CR conditions in polar coordinates to show that $u(r, \theta)$ satisfies

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$$

which is Laplace's equation in polar coordinates. Also show that $v(r, \theta)$ satisfies Laplace's equation in polar coordinates.