Solutions - Section 29

(6)

$$\begin{aligned}
|e^{z^2}| &= |e^{(x+iy)^2}| \\
&= |e^{x^2-y^2+i2xy}| \\
&= |e^{x^2}||e^{-y^2}||e^{i2xy}| \\
&= e^{x^2}e^{-y^2} \\
&< e^{x^2}e^{y^2} \\
&= |e^{|z|^2}|
\end{aligned}$$

Note that $e^{-y^2} < e^{y^2}$ because $-y^2 < y^2$ and the real exponential function is increasing.

(10a) Assume e^z is real. $e^z = e^x \cos y + ie^x \sin y$, so if e^z is real then $e^x \sin y = 0$. e^x is never 0, so $\sin y = 0$ which happens when $y = \operatorname{Im} z$ is a multiple of π.

(10b) Assume e^z is pure imaginary. Then similarly to (10a) we know that $\cos y = 0$. This happens when $y = \operatorname{Im} z = (n + \frac{1}{2})\pi, \quad (n = 0, \pm 1, \pm 2, \ldots)$.