Solutions - Section 32

(1a)

$$\begin{aligned}\log(1+i) &= \log(\sqrt{2}e^{i\frac{\pi}{4}}) \\ &= \frac{1}{2}\ln 2 + i(\frac{\pi}{4} + 2\pi n) \\ (1+i)^i &= e^{i\log(1+i)} \\ &= e^{i\frac{1}{2}\ln 2 - \frac{\pi}{4} - 2\pi n} \\ &= e^{i\frac{1}{2}\ln 2} e^{-\frac{\pi}{4} + 2\pi n}\end{aligned}$$

Note that we were able to switch $-2\pi n$ to $2\pi n$ because n can be any integer, positive or negative. $-n$ gives all the integers, same as n.

(1b)

$$\begin{aligned}\log(-1) &= \ln 1 + i(\pi + 2\pi n) \\ &= i(\pi + 2\pi n) \\ (-1)^{\frac{1}{\pi}} &= e^{\frac{1}{\pi}\log(-1)} \\ &= e^{i + i2n}\end{aligned}$$

(3)

$$\begin{aligned}\log(-1+i\sqrt{3}) &= \log(2e^{i\frac{2\pi}{3}}) \\ &= \ln 2 + i(\frac{2\pi}{3} + 2\pi n) \\ (-1+i\sqrt{3})^{\frac{3}{2}} &= e^{\frac{3}{2}\log(-1+i\sqrt{3})} \\ &= e^{\frac{3}{2}\ln 2 + i\pi + i3\pi n} \\ &= e^{\ln 2^{\frac{3}{2}}} e^{i\pi} e^{i3\pi n} \\ &= (2^{\frac{3}{2}})(-1)(\pm 1) \\ &= \pm 2\sqrt{2}\end{aligned}$$