Solutions - Sections 41, 42

(1a)

$$\begin{aligned}
\int_C f(z)dz &= \int_C \frac{z+2}{z}dz \\
&= \int_0^\pi \frac{2e^{i\theta}+2}{2e^{i\theta}} 2ie^{i\theta}d\theta \\
&= \int_0^\pi 2ie^{i\theta} + 2i\, d\theta \\
&= (2e^{i\theta} + 2i\theta|_0^\pi \\
&= 2e^{i\pi} + 2i\pi - 2 - 0 \\
&= -4 + 2\pi i
\end{aligned}$$

(3) Parameterize the square in four pieces.

- From 0 to 1, the curve C_1 is $z(t) = t$, $0 \le t \le 1$.
- From 1 to $1+i$, the curve C_2 is $z(t) = 1 + it$, $0 \le t \le 1$.
- From $1+i$ to i, the curve C_3 is $z(t) = 1 + i - t$, $0 \le t \le 1$.
- From i to 0, the curve C_4 is $z(t) = i - it$, $0 \le t \le 1$.

$$\begin{aligned}
\int_C f(z)dz &= \int_{C_1} f(z)dz + \int_{C_2} f(z)dz + \int_{C_3} f(z)dz + \int_{C_4} f(z)dz \\
\int_{C_1} \pi e^{\pi\bar{z}}dz &= \int_0^1 \pi e^{\pi t}1dt \\
&= e^{\pi t}|_0^1 \\
&= e^\pi - 1 \\
\int_{C_2} \pi e^{\pi\bar{z}}dz &= \int_0^1 \pi e^{\pi(1-it}i\, dt \\
&= -e^{\pi(1-it)}|_0^1 \\
&= e^\pi + e^\pi \\
\int_{C_3} \pi e^{\pi\bar{z}}dz &= \int_0^1 \pi e^{\pi(1-i-t)}(-1)dt \\
&= e^{\pi(1-i-t)}|_0^1 \\
&= -1 + e^\pi \\
\int_{C_4} \pi e^{\pi\bar{z}}dz &= \int_0^1 \pi e^{\pi(-i+it)}(-i)dt \\
&= -e^{\pi(-i+it)}|_0^1 \\
&= -1 - 1 \\
\int_C f(z)dz &= (e^\pi - 1) + (2e^\pi) + (-1 + e^\pi) + (-2) \\
&= 4e^\pi - 4
\end{aligned}$$