Solutions - Section 54

(3) Let a function f be continuous on a closed bounded region R, and let it be analytic and not constant throughout the interior of R. Assume $f(z) \neq 0$ anywhere in R. Then $g(z) = \frac{1}{f(z)}$ is also analytic (and not constant) throughout R. By the Maximum Modulus Principle, $|g(z)|$ cannot have a maximum value in the interior of R; its maximum occurs on the boundary. But a maximum of $|g(z)|$ is a minimum of $|f(z)|$. So $|f(z)|$ has a minimum value on the boundary of R and not in the interior of R.