1.5 - 1.8 Homework Problems

(5.1) Use properties of conjugates and moduli to show the equality

(a) \( \overline{z} + 3i = z - 3i \)  
(b) \( i\overline{z} = -i\overline{z} \)  
(c) \( (2 + i)^2 = 3 - 4i \)

(5.2) Sketch the set of points determined by \( \text{Re} (\overline{z} - i) = 2 \).

(5.3) Verify the two properties of complex conjugates

\[ \overline{z_1 z_2} = \overline{z_1} \overline{z_2}, \quad \frac{z_1}{z_2} = \frac{\overline{z_1}}{\overline{z_2}} \]

(5.7) Show that \( |\text{Re} (2 + \overline{z} + z^3)| \leq 4 \) when \( |z| \leq 1 \).

(5.9) Show that if \( z \) lies on the circle \( |z| = 2 \), then

\[ \left| \frac{1}{z^4 - 4z^2 + 3} \right| \leq \frac{1}{3} \]

(5.10a) Prove that \( z \) is real if and only if \( \overline{z} = z \).

(8.2a) Show that \( |e^{i\theta}| = 1 \)

(8.2b) Show that \( e^{i\theta} = e^{-i\theta} \)

(8.4) Using the fact that \( |e^{i\theta} - 1| \) gives the distance between \( e^{i\theta} \) and 1, give a geometric argument to find a value of \( \theta \) in the interval \( 0 \leq \theta < 2\pi \) that satisfies the equation \( |e^{i\theta} - 1| = 2 \).

(8.5a) Show the following equality by switching to exponential form.

\[ i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + i\sqrt{3}) \]

(8.5b) Show the following equality by switching to exponential form.

\[ (-1 + i)^7 = 8(-1 - i) \]

(8.6) Show that if \( \text{Re} z_1 > 0 \) and \( \text{Re} z_2 > 0 \), then \( \text{Arg} z_1 z_2 = \text{Arg} z_1 + \text{Arg} z_2 \) (remember \( \text{Arg} \) refers to the principal argument).

(8.10) Use de Moivre’s formula to derive the triple angle formulas.

\[ \cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta, \quad \sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta \]