1.5 - 1.8 Homework Problems

(5.1) Use properties of conjugates and moduli to show the equality

(a) $\overline{\bar{z}+3i} = z - 3i$ (b) $\overline{iz} = -i\bar{z}$ (c) $\overline{(2+i)^2} = 3 - 4i$

(5.2) Sketch the set of points determined by $\mathrm{Re}\,(\bar{z} - i) = 2$.

(5.3) Verify the two properties of complex conjugates

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2, \qquad \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

(5.7) Show that $|\mathrm{Re}\,(2 + \bar{z} + z^3)| \leq 4$ when $|z| \leq 1$.

(5.9) Show that if z lies on the circle $|z| = 2$, then

$$\left|\frac{1}{z^4 - 4z^2 + 3}\right| \leq \frac{1}{3}$$

(5.10a) Prove that z is real if and only if $\bar{z} = z$.

(8.2a) Show that $|e^{i\theta}| = 1$

(8.2b) Show that $\overline{e^{i\theta}} = e^{-i\theta}$

(8.4) Using the fact that $|e^{i\theta} - 1|$ gives the distance between $e^{i\theta}$ and 1, give a geometric argument to find a value of θ in the interval $0 \leq \theta < 2\pi$ that satisfies the equation $|e^{i\theta} - 1| = 2$.

(8.5a) Show the following equality by switching to exponential form.

$$i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + i\sqrt{3})$$

(8.5b) Show the following equality by switching to exponential form.

$$(-1 + i)^7 = 8(-1 - i)$$

(8.6) Show that if $\mathrm{Re}\, z_1 > 0$ and $\mathrm{Re}\, z_2 > 0$, then $\mathrm{Arg}\, z_1 z_2 = \mathrm{Arg}\, z_1 + \mathrm{Arg}\, z_2$ (remember Arg refers to the principal argument).

(8.10) Use de Moivre's formula to derive the triple angle formulas.

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta, \qquad \sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$