

Problems - Sections 75, 76

(3a) Show that

$$\operatorname{Res}_{z_n} z \sec z = (-1)^{n+1} z_n$$

where $z_n = \frac{\pi}{2} + n\pi$ for n any integer.

(4a) Let C be the circle $|z| = 2$ positively oriented. Evaluate

$$\int_C \tan z \, dz$$

(7) Consider the function

$$f(z) = \frac{1}{(q(z))^2}$$

where q is analytic at z_0 , $q(z_0) = 0$ and $q'(z_0) \neq 0$. Show that z_0 is a pole of order 2 of the function f , with residue

$$\operatorname{Res}_{z_0} f(z) = -\frac{q''(z_0)}{(q'(z_0))^3}$$