Problems - Sections 82, 83, 84

(4) Use $f(z) = \frac{(\log z)^2}{z^2+1}$ to show that

$$\int_0^\infty \frac{(\ln x)^2}{x^2+1}dx = \frac{\pi^3}{8}, \qquad \int_0^\infty \frac{\ln x}{x^2+1}dx = 0$$

(One integral comes from the real part, one from the imaginary part of the complex integral.)

(6a) Evaluate the integral $\int_0^\infty \frac{x^{-\frac{1}{2}}}{x^2+1}dx$ using an upper-half circle contour with an indent around the branch cut. That is, go from ρ to R, around the half circle C_R to $-R$, up to $-\rho$, then around C_ρ back to ρ.

(6b) Evaluate the integral $\int_0^\infty \frac{x^{-\frac{1}{2}}}{x^2+1}dx$ using a full circle "pacman" contour along the branch cut. That is, go from ρ to R on top of the cut, around the full circle C_R to R under the cut, back to ρ under the cut, then around C_ρ to ρ on top of the cut.

(D) Evaluate the integral

$$\int_0^\infty \frac{1}{x^a(x+1)}dx$$

for $0 < a < 1$, using the method from (6b).