1.1 Homework Solutions

(14) Show that "FOILing" works. That is,

$$(a+b)(c+d) = ac + ad + bc + bd.$$

$$\begin{aligned}
(a+b)(c+d) &= (a+b)c + (a+b)d \qquad (D)\\
&= c(a+b) + d(a+b) \qquad (M2)\\
&= ca + cb + da + db \qquad (D)\\
&= ac + bc + ad + bd \qquad (M2)\\
&= ac + ad + bc + bd \qquad (A2)
\end{aligned}$$

(15) Prove that if $b \neq 0$ and $c \neq 0$, then

$$\frac{a}{b} = \frac{ac}{bc}.$$

$$\begin{aligned}
\frac{ac}{bc} &= (ac)(bc)^{-1}\\
&= (ac)(b^{-1}c^{-1}) \qquad (1.6)\\
&= (ac)(c^{-1}b^{-1}) \qquad (M2)\\
&= (a(c \cdot c^{-1}))b^{-1} \qquad (M3 \quad twice)\\
&= (a1)b^{-1} \qquad (M5)\\
&= a \cdot b^{-1} \qquad (M4)\\
&= \frac{a}{b}
\end{aligned}$$

(16) Prove that if $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}.$$

$$\begin{aligned}
\frac{a+b}{c} &= (a+b)c^{-1}\\
&= a \cdot c^{-1} + b \cdot c^{-1} \qquad (D)\\
&= \frac{a}{c} + \frac{b}{c}
\end{aligned}$$

(19) Prove that if $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}.$$

$$
\begin{aligned}
\frac{ad+bc}{bd} &= (ad+bc)(bd)^{-1} \\
&= (ad)(bd)^{-1} + (bc)(bd)^{-1} \qquad (D) \\
&= (ad)(b^{-1}d^{-1}) + (bc)(b^{-1}d^{-1}) \qquad (1.6) \\
&= (ad)(d^{-1}b^{-1}) + (cb)(b^{-1}d^{-1}) \qquad (M2) \\
&= (a(d \cdot d^{-1}))b^{-1} + (c(b \cdot b^{-1}))d^{-1} \qquad (M3 \quad twice) \\
&= (a1)b^{-1} + (c1)d^{-1} \qquad (M5) \\
&= a \cdot b^{-1} + c \cdot d^{-1} \qquad (M4) \\
&= \frac{a}{b} + \frac{c}{d}
\end{aligned}
$$