1.3 Homework Solutions

(17) Find the solution set of the inequality

$$\frac{x+2}{x-1} < 4$$

The solution set $S = S_1 \cup S_2$, where $S_1 = \{x : \frac{x+2}{x-1} < 4 \text{ and } x-1>0\}$ and $S_2 = \{x : \frac{x+2}{x-1} < 4 \text{ and } x-1<0\}$.

(S_1)

$$\begin{aligned} \frac{x+2}{x-1} &< 4 \quad \textit{and} \quad x-1>0 \\ x+2 &< 4(x-1) \quad \textit{and} \quad x-1>0 \\ x+2 &< 4x-4 \quad \textit{and} \quad x>1 \\ 6 &< 3x \quad \textit{and} \quad x>1 \\ x &> 2 \quad \textit{and} \quad x>1 \end{aligned}$$

S_1 must satisfy both inequalities, so it is the set $(2, \infty)$.

(S_2)

$$\begin{aligned} \frac{x+2}{x-1} &< 4 \quad \textit{and} \quad x-1<0 \\ x+2 &> 4(x-1) \quad \textit{and} \quad x-1<0 \\ x+2 &> 4x-4 \quad \textit{and} \quad x<1 \\ 6 &> 3x \quad \textit{and} \quad x<1 \\ x &< 2 \quad \textit{and} \quad x<1 \end{aligned}$$

S_1 must satisfy both inequalities, so it is the set $(-\infty, 1)$.

$S = S_1 \cup S_2 = (-\infty, 1) \cup (2, \infty)$.

(20) Prove Theorem 1.13(ii) - If $a < 0$ and $b < 0$, then $ab > 0$.

$$\begin{aligned} a<0, \quad b<0 &\Rightarrow a, b \text{ negative} \\ &\Rightarrow -a, -b \text{ positive} \\ &\Rightarrow (-a)(-b) = ab \text{ positive, by } I \\ &\Rightarrow ab > 0 \end{aligned}$$

(22) Prove Theorem 1.15(iii) - If $a > b$ and a and b have the same sign, then $\frac{1}{a} < \frac{1}{b}$.

First, note that since a, b have signs, they are non-zero by definition. Since a, b have the same sign, $ab > 0$. By 1.15(i), $\frac{a}{ab} > \frac{b}{ab}$. But $\frac{a}{ab} = \frac{1}{b}$ and $\frac{b}{ab} = \frac{1}{a}$, so $\frac{1}{b} > \frac{1}{a}$ and $\frac{1}{a} < \frac{1}{b}$.

(24) Prove - If $a > b$ and $c > d$, then $a + c > b + d$.

$$\begin{aligned} a > b, \quad c > d \ &\Rightarrow\ a - b, c - d \text{ positive} \\ &\Rightarrow\ (a - b) + (c - d) = (a + c) - (b + d) \text{ positive} \\ &\Rightarrow\ a + c > b + d \end{aligned}$$