1.4 Homework Solutions

(3) Use induction to establish the formula

$$\sum_{i=1}^{n}(2i-1) = n^2$$

Let S be the set of natural numbers for which the formula holds. We will show that S is inductive, so that $\mathbb{N} \in S$.

Check $n = 1$. $(2(1) - 1) = 2 - 1 = 1 = 1^2$. So $1 \in S$.

Suppose $k \in S$.

$$\begin{aligned}
\sum_{i=1}^{k+1}(2i-1) &= (2(k+1)-1) + \sum_{i=1}^{k}(2i-1) \\
&= (2k+2-1) + (k^2) \\
&= k^2 + 2k + 1 \\
&= (k+1)^2
\end{aligned}$$

So $k + 1 \in S$. Therefore S is inductive, $\mathbb{N} \in S$, and the formula holds for $\mathbb{N}$.

(6) Use induction to establish the formula

$$\sum_{i=1}^{n}\frac{1}{i(i+1)} = \frac{n}{n+1}$$

Let S be the set of natural numbers for which the formula holds. We will show that S is inductive, so that $\mathbb{N} \in S$.

Check $n = 1$. $\frac{1}{1(1+1)} = \frac{1}{1+1}$. So $1 \in S$.

Suppose $k \in S$.

$$\begin{aligned}
\sum_{i=1}^{k+1}\frac{1}{i(i+1)} &= \frac{1}{(k+1)(k+2)} + \sum_{i=1}^{k}\frac{1}{i(i+1)} \\
&= \frac{1}{(k+1)(k+2)} + \frac{k}{k+1} \\
&= \frac{1+k(k+2)}{(k+1)(k+2)} \\
&= \frac{k^2+2k+1}{(k+1)(k+2)} \\
&= \frac{(k+1)^2}{(k+1)(k+2)} \\
&= \frac{k+1}{k+2}
\end{aligned}$$

So $k+1 \in S$. Therefore S is inductive, $\mathbb{N} \in S$, and the formula holds for $\mathbb{N}$.

(13) Use induction to establish the assertion

$$(1+a)^n \geq 1+na \qquad \text{for } a \geq 0 \text{ and } n \in \mathbb{N}$$

Let S be the set of natural numbers for which the assertion holds. We will show that S is inductive, so that $\mathbb{N} \in S$.

Check $n=1$. $(1+a)^1 = 1+a = 1+(1)a$. So $1 \in S$.

Suppose $k \in S$.

$$\begin{aligned}(1+a)^{k+1} &= (1+a)^k(1+a) \\ &\geq (1+ka)(1+a) \\ &\geq 1+(k+1)a+ka^2 \\ &\geq 1+(k+1)a\end{aligned}$$

So $k+1 \in S$. Therefore S is inductive, $\mathbb{N} \in S$, and the assertion holds for $\mathbb{N}$.