Solutions - 2.1/2.2 Limits

(2) Find a δ so that $|x-2| < \delta \Rightarrow |(1-3x)-(-5)| < 0.01$.

Assume $|x-2| < \delta$.

$$\begin{aligned} |(1-3x)-(-5)| &= |6-3x| \\ &= |-3(x-2)| \\ &= 3|x-2| \\ &\leq 3\delta \end{aligned}$$

As long as $|x-2| < \delta = \frac{0.01}{3}$, then $|(1-3x)-(-5)| < 0.01$.

(4) Find a δ so that $|x-1| < \delta \Rightarrow |\sqrt[3]{x}-1| < 0.1$.

Assume $|x-1| < \delta$.

$$\begin{aligned} |x^{\frac{1}{3}}-1| &= \frac{|x^{\frac{1}{3}}-1||x^{\frac{2}{3}}+x^{\frac{1}{3}}+1|}{|x^{\frac{2}{3}}+x^{\frac{1}{3}}+1|} \\ &= \frac{|x-1|}{|x^{\frac{2}{3}}+x^{\frac{1}{3}}+1|} \quad \text{as long as } \delta \leq 1, \quad x \geq 0 \\ &\leq \frac{|x-1|}{0+0+1} \\ &= |x-1| \\ &< \delta \end{aligned}$$

As long as $|x-1| < \delta = 0.1$, then $|\sqrt[3]{x}-1| < 0.1$.

(6) Suppose $\epsilon > 0$. Find a δ so that $|x-1| < \delta \Rightarrow |(x^3-4)-(-3)| < \epsilon$.

Assume $|x-1| < \delta$.

$$\begin{aligned} |(x^3-4)-(-3)| &= |x^3-1| \\ &= |x-1||x^2+x+1| \quad \text{as long as } \delta \leq 1, \quad x \leq 2 \\ &< \delta(4+2+1) \\ &\leq 7\delta \end{aligned}$$

As long as $|x-2| < \delta = \min\{1, \frac{\epsilon}{7}\}$, then $|(x^3-4)-(-3)| < \epsilon$.