2.1/2.2 – Limits and Continuity – Extra Limit Practice

Verify the following limits using ε's and δ's.

(1)

$$\lim_{x\to 2}(x^2-1)=3$$

Let $\varepsilon > 0$ be given. If $|x-2| < \delta = \min\left\{1, \frac{\varepsilon}{5}\right\}$

$$\begin{aligned}|x^2-1-3| &= |x^2-4| \\ &= |x-2||x+2| \\ &< \delta(5) \le \frac{\varepsilon}{5}(5) = \varepsilon\end{aligned}$$

(2)

$$\lim_{x\to 3}(2x+5)=11$$

Let $\varepsilon > 0$ be given. If $|x-3| < \delta = \frac{\varepsilon}{2}$

$$\begin{aligned}|2x+5-11| &= |2x-6| \\ &= 2|x-3| \\ &< 2\delta = 2\frac{\varepsilon}{2} = \varepsilon\end{aligned}$$

(3)

$$\lim_{x\to -1}(5-2x)=7$$

Let $\varepsilon > 0$ be given. If $|x-(-1)| < \delta = \frac{\varepsilon}{2}$

$$\begin{aligned}|5-2x-7| &= |-2x-2| \\ &= |-2||x+1| \\ &< 2\delta = 2\frac{\varepsilon}{2} = \varepsilon\end{aligned}$$

(4)

$$\lim_{x\to 3}\frac{1}{x^2}=\frac{1}{9}$$

Let $\varepsilon > 0$ be given. If $|x-3| < \delta = \min\left\{1, \frac{36\varepsilon}{7}\right\}$

$$\left|\frac{1}{x^2}-\frac{1}{9}\right| = \left|\frac{9-x^2}{9x^2}\right|$$

$$= \frac{|3-x||3+x|}{9|x||x|}$$

Since $\delta \leq 1$,

$$|x-3| < 1$$
$$-1 < x-3 < 1$$
$$2 < x < 4$$
$$5 < x < 7$$

So

$$\left|\frac{1}{x^2} - \frac{1}{9}\right| = \frac{|3-x||3+x|}{9|x||x|}$$
$$< \frac{\delta 7}{9(2)(2)}$$
$$= \frac{7\delta}{36} \leq \frac{7\delta}{36}\frac{36\varepsilon}{7} = \varepsilon$$