

Problems - 2.5 - Limits of Sequences

(1) Given the function $f(x) = x \cos x$.

(a) Find a sequence of numbers $\{x_n\}$ such that

$$\lim_{n \rightarrow \infty} x_n = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} f(x_n) = 0$$

(b) Find a sequence of numbers $\{x_n\}$ such that

$$\lim_{n \rightarrow \infty} x_n = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} f(x_n) = +\infty$$

(c) Find a sequence of numbers $\{x_n\}$ such that

$$\lim_{n \rightarrow \infty} x_n = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} f(x_n) = -\infty$$

(6) If $-1 < a < 1$, show that $\lim_{n \rightarrow \infty} a^n = 0$.