Solutions - 3.3

(2) Decide whether or not the sequence converges to a limit. If it does not, find a convergent subsequence. (You do not need to check the limits with ϵ and δ.)

$$x_n = 1 + \frac{(-1)^n}{n}$$

The term $\frac{(-1)^n}{n}$ goes to 0 as n goes to infinity; it would be a short proof similar to Th 2.21(c). This means

$$\lim_{n\to\infty} x_n = 1 + 0 = 1$$

(6) Decide whether or not the sequence converges to a limit. If it does not, find a convergent subsequence. (You do not need to check the limits with ϵ and δ.)

$$x_n = \sin(\frac{n\pi}{2}) + \cos(n\pi)$$

$\sin(\frac{n\pi}{2})$ cycles through the pattern $1, 0, -1, 0, 1, 0, -1, 0, \ldots$. Meanwhile, $\cos(n\pi)$ follows the pattern $-1, 1, -1, 1, \ldots$. Together, we get

$$1-1, 0+1, -1-1, 0+1, 1-1, 0+1, -1-1, 0+1, \ldots = 0, 1, -2, 1, 0, 1, -2, 1, \ldots$$

This sequence does not converge to any limit. However, there are many choices for subsequences. If we start with any term, then take every fourth term after that, our subsequence will be constant (and therefore convergent). For example,

$$x_{4n} = \sin(\frac{4n\pi}{2}) + \cos(4n\pi) = 0 + 1 = 1 \qquad \forall \qquad n$$