Solutions - 3.4

(0) Suppose that the domain of f is the interval $I = [0, 5)$ and f is continuous on I. Give an example to show that the conclusion of the Boundedness Theorem does not hold.

The function $f(x) = \frac{1}{x-5}$ is continuous except at $x = 5$, which is not in I, so f is continuous on I. Not that $x_n = 5 + \frac{1}{n}$ is in I for each natural number n, but $f(x_n) = n$. Therefore f is unbounded on I.

(3) Suppose that f is continuous on the set $S = [0, \infty)$ and S is bounded. Give an example to show that the conclusion of the Extreme Value Theorem does not hold.

Let $f(x) = \frac{1}{x^2+1}$. Since the denominator is not 0 for any real numbers, f is continuous. $f(x) \neq 0$ for any real number, but for any number $0 < c \leq 1$, $f(\sqrt{\frac{1}{c} - 1}) = c$. So $\inf_S f(x) = 0$ but $f(x) \neq 0$ for any number in S.