

### Sequence examples - Chapter 3

For each set of conditions, find a sequence or function which satisfies the conditions (give a formula OR a graph OR a detailed description). If no such sequence or function exists, give a theorem which shows that it can't exist.

- (1) A sequence which is bounded but does not converge.
- (2) A sequence with no convergent subsequence.
- (3) A bounded sequence with no convergent subsequence.
- (4) A sequence which is unbounded and has a convergent subsequence.
- (5) A sequence with subsequences which converge to two different numbers.
- (6) A function which is continuous and unbounded on the interval  $(1, 2)$ .
- (7) A function which achieves its sup and inf on the interval  $(-2, 2)$ . (The conclusion of EVT holds for this example.)
- (8) A function which is uniformly continuous and unbounded.
- (9) A function continuous on  $[0, 2]$  with range equal to  $[0, 2) \cup (3, 5]$ .
- (10) A function which is uniformly continuous, bounded and  $\lim_{x \rightarrow +\infty} f(x)$  does not exist.